Web Appendices for
“The Technology of Skill Formation”

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A Deriving the Technology of Skill Formation from its Primitives

A Model of Skill Formation

In the models presented in this section of the appendix and in CHLM, parents make decisions about their children. We ignore how the parents get to be who they are and the decisions of the children about their own children. Appendix C develops a generationally consistent model.

Suppose that there are two periods in a child’s life, “1” and “2”, before the child becomes an adult. Adulthood comprises distinct third and fourth periods. The child works for two periods after the two periods of childhood. Models based on the analysis of Becker and Tomes (1986) assume only one period of childhood. We assume that there are two kinds of skills: $\theta_C$ and $\theta_N$.

For example, $\theta_C$ can be thought of as cognitive skill and $\theta_N$ as noncognitive skill. Our treatment of ability is in contrast to the view of the traditional literature on human capital formation that views IQ as innate ability. In our analysis, IQ is just another skill. What differentiates IQ from other cognitive and noncognitive skills is that IQ is subject to accumulation during critical periods. That is, parental and social interventions can increase the IQ of the child, but they can do so successfully only for a limited time.

Let $I_k^t$ denote parental investments in child skill $k$ at period $t, k = C, N$ and $t = 1, 2$. Let $h'$ be the level of human capital as the child starts adulthood. It depends on both components of $(\theta^C_2, \theta^N_2)$. The parents fully control the investment of the child. A richer model incorporates, among other features, investment decisions of the child as influenced by the parent through preference formation processes (see Carneiro, Cunha, and Heckman, 2003, and Cunha and Heckman, 2007a).

We first describe how skills evolve over time. Assume that each agent is born with initial conditions $\theta_1 = (\theta^C_1, \theta^N_1)$. These can be determined by parental environments. At each stage $t$ let $\theta_t = (\theta^C_t, \theta^N_t)$ denote the vector of skill or ability stocks. The technology of production of skill $k$ at period $t$ is

$$\theta^k_{t+1} = f^k_t(\theta_t, I^k_t),$$  \hspace{1cm} (A-1)

for $k = C, N$ and $t = 1, 2$. We assume that $f^k_t$ is twice continuously differentiable, increasing and concave. In this model, stocks of both skills and abilities produce next period skills and the productivity of investments. Cognitive skills can promote the formation of noncognitive skills and vice
versa.

Let \( \theta^C_3, \theta^N_3 \) denote the level of skills when adult. We define adult human capital \( h' \) of the child as a combination of different adult skills:

\[
h' = g \left( \theta^C_3, \theta^N_3 \right).
\] (A-2)

The function \( g \) is assumed to be continuously differentiable and increasing in \( \left( \theta^C_3, \theta^N_3 \right) \). This model assumes that there is no comparative advantage in the labor market or in life itself. \(^1\)

To fix ideas, consider the following specialization of our model. Ignore the effect of initial conditions and assume that first period skills are just due to first period investment:

\[
\theta^C_2 = f^C_1 \left( \theta_1, I^C_1 \right) = I^C_1
\]

and

\[
\theta^N_2 = f^N_1 \left( \theta_1, I^C_1 \right) = I^N_1,
\]

where \( I^C_1 \) and \( I^N_1 \) are scalars. For the second period technologies, assume a CES structure:

\[
\theta^C_3 = f^C_2 \left( \theta_2, I^C_2 \right) = \left\{ \gamma_1 \left( \theta^C_2 \right)^\alpha + \gamma_2 \left( \theta^N_2 \right)^\alpha + \left( 1 - \gamma_1 - \gamma_2 \right) \left( I^C_2 \right)^\alpha \right\}^{\frac{1}{\alpha}} \quad \text{where} \quad 1 \geq \gamma_1 \geq 0, \quad 1 \geq \gamma_2 \geq 0, \quad 1 \geq 1 - \gamma_1 - \gamma_2 \geq 0,
\] (A-3)

and

\[
\theta^N_3 = f^N_2 \left( \theta_2, I^N_2 \right) = \left\{ \eta_1 \left( \theta^C_2 \right)^\nu + \eta_2 \left( \theta^N_2 \right)^\nu + \left( 1 - \eta_1 - \eta_2 \right) \left( I^N_2 \right)^\nu \right\}^{\frac{1}{\nu}} \quad \text{where} \quad 1 \geq \eta_1 \geq 0, \quad 1 \geq \eta_2 \geq 0, \quad 1 \geq 1 - \eta_1 - \eta_2 \geq 0,
\] (A-4)

where \( \frac{1}{1-\alpha} \) is the elasticity of substitution in the inputs producing \( \theta^C_3 \) and \( \frac{1}{1-\nu} \) is the elasticity of substitution of inputs in producing \( \theta^N_3 \) where \( \alpha \in (-\infty, 1] \) and \( \nu \in (-\infty, 1] \). Notice that \( I^N_2 \) and \( I^C_2 \) are direct complements with \( (\theta^C_2, \theta^N_2) \)

\(^1\)Thus we rule out one potentially important avenue of compensation that agents can specialize in tasks that do not require the skills in which they are deficient. In CHLM we briefly consider a more general task function that captures the notion that different tasks require different combinations of skills and abilities.
irrespective of the substitution parameters $\alpha$ and $\nu$, except in limiting cases.

The CES technology is well known and has convenient properties. It imposes direct complementarity even though inputs may be more or less substitutable depending on $\alpha$ or $\nu$. We distinguish between direct complementarity (positive cross partials) and CES-substitution/complementarity. Focusing on the technology for producing $\theta_C^3$, when $\alpha = 1$, the inputs are perfect substitutes in the intuitive use of that term (the elasticity of substitution is infinite). The inputs $\theta_C^2$, $\theta_N^2$ and $I_C^2$ can be ordered by their relative productivity in producing $\theta_C^3$. The higher $\gamma_1$ and $\gamma_2$, the higher the productivity of $\theta_C^2$ and $\theta_N^2$ respectively. When $\alpha = -\infty$, the elasticity of substitution is zero. All inputs are required in the same proportion to produce a given level of output so there are no possibilities for technical substitution, and

$$\theta_C^3 = \min\{\theta_C^2, \theta_N^2, I_C^2\}.$$

In this technology, early investments are a bottleneck for later investment. Compensation for adverse early environments through late investments is impossible.

The evidence from numerous studies reviewed in the text and in CHLM cited shows that IQ is no longer malleable after ages 8-10. Taken at face value, this implies that if $\theta_C$ is IQ, for all values of $I_C^2$, $\theta_C^3 = \theta_C^2$. Period 1 is a critical period for IQ but not necessarily for other skills and abilities. More generally, period 1 is a critical period if

$$\frac{\partial \theta_C^{t+1}}{\partial I_C^t} = 0 \text{ for } t > 1.$$

For parameterization (A-3), this is obtained by imposing $\gamma_1 + \gamma_2 = 1$.

The evidence on adolescent interventions surveyed in CHLM shows substantial positive results for such interventions on noncognitive skills ($\theta_N^3$) and at most modest gains for cognitive skills. Technologies (A-3) and (A-4) can rationalize this pattern. Since the populations targeted by adolescent intervention studies tend to come from families with poor backgrounds, we would expect $I_C^1$ and $I_N^1$ to be below average. Thus, $\theta_C^2$ and $\theta_N^2$ will be below average. Adolescent interventions make $I_C^2$ and $I_N^2$ relatively large for the treatment group in comparison to the control group in the intervention experiments. At stage 2, $\theta_C^3$ (cognitive ability) is essentially the same in the control and treatment groups, while $\theta_N^3$ (noncognitive ability) is higher for the treated group. Large values of $(\gamma_1 + \gamma_2)$ (associated with a small coefficient on $I_C^2$) or small values of $(\eta_1 + \eta_2)$ (so the coefficient on $I_N^2$ is
large) and high values of $\alpha$ and $\nu$ can produce this pattern. Another case that rationalizes the evidence is when $\alpha \rightarrow -\infty$ and $\nu = 1$. Under these conditions,

$$\theta_3^C = \min \{ \theta_2^C, \theta_2^N, I_2^C \}, \quad (A-5)$$

while

$$\theta_3^N = \eta_1 \theta_2^C + \eta_2 \theta_2^N + (1 - \eta_1 - \eta_2) I_2^N. \quad (A-6)$$

The attainable period 2 stock of cognitive skill ($\theta_3^C$) is limited by the minimum value of $\theta_2^C, \theta_2^N, I_2^C$. In this case, any level of investment in period 2 such that $I_2^C > \min \{ \theta_2^C, \theta_2^N \}$ is ineffective in incrementing the stock of cognitive skills. Period 1 is a bottleneck period. Unless sufficient skill investments are made in $\theta_C$ in period 1, it is not possible to raise skill $\theta_C$ in period 2. This phenomenon does not appear in the production of the noncognitive skill, provided that $(1 - \eta_1 - \eta_2) > 0$. More generally, the higher $\nu$ and the larger $(1 - \eta_1 - \eta_2)$, the more productive is investment $I_2^N$ in producing $\theta_2^N$.

To complete the CES example, assume that adult human capital $h'$ is a CES function of the two skills accumulated at stage two:

$$h' = \left\{ \tau \left( \theta_3^C \right)^\nu + (1 - \tau) \left( \theta_3^N \right)^\nu \right\}^{\frac{1}{1-\phi}}, \quad (A-7)$$

where $\rho \in (0, 1)$, $\tau \in [0, 1]$, and $\phi \in (-\infty, 1]$. In this parameterization, $\frac{1}{1-\phi}$ is the elasticity of substitution across different skills in the production of adult human capital. Equation (A-7) reminds us that the market, or life in general, requires use of multiple skills. Being smart isn’t the sole determinant of success. In general, different tasks require both skills in different proportions. One way to remedy early skill deficits is to make compensatory investments. Another way is to motivate people from disadvantaged environments to pursue tasks that do not require the skill that deprived early environments do not produce. A richer theory would account for this choice of tasks and its implications for remediation. \(^2\) For the sake of simplifying our argument, we work with equation (A-7) that captures the notion that skills can trade off against each other in producing effective people. Highly motivated, but not very bright, people may be just as effective as bright but unmotivated people. That is one of the lessons from the GED program. (See Heckman and Rubinstein, 2001, and Heckman, Stixrud, and Urzua, 2006.)

The analysis is simplified by assuming that investments are general in

\(^2\)See the appendix in CHLM.
nature: $I^C_1 = I^N_1 = I_1$, $I^C_2 = I^N_2 = I_2$. Cunha and Heckman (2007a,b) develop the more general case of skill-specific investments which requires more notational complexity.

With common investment goods, we can solve out for $\theta^C_2$ and $\theta^N_2$ in terms of $I_1$ to simplify (A-3) and (A-4) to reach

$$\theta^C_2 = \left( (\gamma_1 + \gamma_2) (I_1)^\alpha + (1 - \gamma_1 - \gamma_2) (I_2)^\alpha \right)^{\frac{1}{\alpha}}$$  \hspace{1cm} (A-8)$$

and

$$\theta^N_2 = \left( (\eta_1 + \eta_2) (I_1)^\nu + (1 - \eta_1 - \eta_2) (I_2)^\nu \right)^{\frac{1}{\nu}}.$$  \hspace{1cm} (A-9)$$

If we then substitute these expressions into the production function for adult human capital (A-7), we obtain

$$h' = \left\{ \tau \left[ \tilde{\gamma} (I_1)^\phi + (1 - \tilde{\gamma}) (I_2)^\phi \right]^{\frac{\phi}{\phi + \alpha}} + (1 - \tau) \left[ \tilde{\eta} (I_1)^\nu + (1 - \tilde{\eta}) (I_2)^\nu \right]^{\frac{\nu}{\nu + \phi}} \right\}^{\frac{1}{\phi + \alpha}}.$$  \hspace{1cm} (A-10)$$

where $\tilde{\gamma} = \gamma_1 + \gamma_2$, $\tilde{\eta} = \eta_1 + \eta_2$. Equation (A-10) expresses adult human capital as a function of the entire sequence of childhood investments in human capital. Current investments in human capital are combined with the existing stocks of skills in order to produce the stock of next period skills.

A conveniently simple formulation of the problem arises if we assume that $\alpha = \nu = \phi$ so that CES substitution among inputs in producing outputs and CES substitution among skill in producing human capital are the same. This produces the convenient and familiar-looking CES expression for adult human capital stocks:

$$h' = \left\{ \gamma I_1^\phi + (1 - \gamma) I_2^\phi \right\}^{\frac{\phi}{\phi + \alpha}}.$$  \hspace{1cm} (A-11)$$

where $\gamma = \tau \tilde{\gamma} + (1 - \tau) \tilde{\eta}$ and $\phi = \alpha = \nu$. The parameter $\gamma$ is a skill multiplier. It arises because $I_1$ affects the accumulation of $\theta^C_2$ and $\theta^N_2$. These stocks of skills in turn affect the productivity of $I_2$ in forming $\theta^C_3$ and $\theta^N_3$. Thus $\gamma$ captures the net effect of $I_1$ on $h'$ through both self-productivity and direct

---

3Thus when a parent buys a book in the first period of childhood, this book may be an investment in all kinds of skills. It is an investment in cognitive skills, as it helps the child get exposure to language and new words. It can also be an investment in noncognitive skills, if the book may contain a message on the importance of being persistent and patient.
complementarity.\(^4\) \(\frac{1}{1-\phi}\) is a measure of how easy it is to substitute between \(I_1\) and \(I_2\) where the substitution arises from both the task performance (human capital) function in equation (A-7) and the technology of skill formation. Within the CES technology, \(\phi\) is a measure of the ease of substitution of inputs. In this analytically convenient case, the parameter \(\phi\) plays a dual role. First, it informs us how easily one can substitute across different skills in order to produce one unit of adult human capital \(h'\). Second, it also represents the degree of complementarity (or substitutability) between early and late investments in producing skills. In this second role, the parameter \(\phi\) dictates how easy it is to compensate for low levels of stage 1 skills in producing late skills.

In principle, compensation can come through two channels: (i) through skill investment or (ii) through choice of market activities, substituting deficits in one skill by the relative abundance in the other through choice of tasks. We do not develop the second channel of compensation in this appendix, deferring it to later work. It is discussed in Carneiro, Cunha, and Heckman (2003).

When \(\phi\) is small, low levels of early investment \(I_1\) are not easily remedi ed by later investment \(I_2\) in producing human capital. The other face of CES complementarity is that when \(\phi\) is small, high early investments should be followed with high late investments. In the extreme case when \(\phi \to -\infty\), (A-11) converges to \(h' = (\min \{I_1, I_2\})^{\rho}\). We analyzed this case in CHLM. The Leontief case contrasts sharply with the case of perfect CES substitutes, which arises when \(\phi = 1\): \(h' = [\gamma I_1 + (1 - \gamma) I_2]^{\rho}\). When we impose the further restriction that \(\gamma = \frac{1}{2}\), we generate the model that is implicitly assumed in the existing literature on human capital investments that collapses childhood into a single period. In this special case, only the total

\[^4\text{Direct complementarity between } I_1 \text{ and } I_2 \text{ arises if}\]

\[
\frac{\partial^2 h}{\partial I_1 \partial I_2} > 0.
\]

As long as \(\rho > \phi\), \(I_1\) and \(I_2\) are direct complements, because

\[
\text{sign}\left(\frac{\partial^2 h}{\partial I_1 \partial I_2}\right) = \text{sign}(\rho - \phi).
\]

This definition of complementarity is to be distinguished from the notion based on the elasticity of substitution between \(I_1\) and \(I_2\), which is \(\frac{1}{1-\phi}\). When \(\phi < 0\), \(I_1\) and \(I_2\) are sometimes called complements. When \(\phi > 0\), \(I_1\) and \(I_2\) are sometimes called substitutes. When \(\rho = 1\), \(I_1\) and \(I_2\) are always direct complements, but if \(1 > \phi > 0\), they are CES substitutes.
amount of human capital investments, regardless of how it is distributed across childhood periods, determines adult human capital. In the case of perfect CES substitutes, it is possible in a physical productivity sense to compensate for early investment deficits by later investments, although it may not be economically efficient to do so.

When \( \rho = 1 \), we can rewrite (A-11) as

\[
h' = I_1 \left\{ \gamma + (1 - \gamma) \omega^\phi \right\}^{\frac{1}{\phi}},
\]

where \( \omega = I_2/I_1 \). Fixing \( I_1 \) (early investment), an increase in \( \omega \) is the same as an increase in \( I_2 \). The marginal productivity of late investment is

\[
\frac{\partial h'}{\partial \omega} = (1 - \gamma) I_1 \left\{ \gamma + (1 - \gamma) \omega^\phi \right\}^{\frac{1-\phi}{\phi}} \omega^{\phi-1}.
\]

For \( \omega > 1 \) and \( \gamma < 1 \), marginal productivity is increasing in \( \phi \) and \( (1 - \gamma) \). Thus, provided that late investments are greater than earlier investments, the more substitutable \( I_2 \) is with \( I_1 \) (the higher \( \phi \)) and the lower the skill multiplier \( \gamma \), the more productive are late investments. Figure A1 graphs the isoquants for \( \frac{\partial h'}{\partial \omega} \) when \( \omega = 2 \). It shows that a high \( \phi \) trades off with a high \( \gamma \). As \( (\phi, 1 - \gamma) \) increases along a ray, \( \frac{\partial h'}{\partial \omega} \) increases. For a fixed skill multiplier \( \gamma \), the higher \( \phi \), the higher the marginal productivity of second period investment.

If, however, \( \omega < 1 \) as in Figure A2, then \( \frac{\partial h'}{\partial \omega} \) could be decreasing as \( (\phi, 1 - \gamma) \) increases along a ray and the trade-off between \( \phi \) and \( (1 - \gamma) \) along a \( \left( \frac{\partial h'}{\partial \omega}, \omega \right) \) isoquant is reversed. If \( I_1 \) is large relative to \( I_2 \) (i.e., \( \omega < 1 \)), for a fixed \( \gamma \) the marginal product of \( I_2 \) is decreasing in \( \phi \). More CES complementarity implies greater productivity (see Figure A2).\(^5\) The empirically relevant case for the analysis of investment in disadvantaged children is \( \omega > 1 \) as shown in Figure C3, so greater CES-substitutability and a smaller skill multiplier produce a higher marginal productivity of remedial second period investment.

\(^5\)One can show that at sufficiently low values of \( \phi \), the marginal productivity is no longer increasing in \( \phi \).
Define $\omega = \frac{I_2}{I_1}$, the ratio of late to early investments in human capital. From the homogeneity of degree one we can rewrite the technology as:

$$h = I_1 \left[ \gamma + (1 - \gamma) \omega^\phi \right]^{\frac{1}{\phi}}.$$

The marginal product of the ratio of late to early investment, $\omega$, holding early investment constant, is

$$\frac{\partial h}{\partial \omega} = (1 - \gamma) I_1 \left[ \gamma + (1 - \gamma) \omega^\phi \right]^{-\frac{1}{\phi}} (1 - \frac{\phi}{\omega^\phi}) \omega^{\phi - 1}.$$

This figure displays the indifference curves of $\frac{\partial h}{\partial \omega}$ when $\omega = 0.5$. Each indifference curve shows the corresponding level of $\frac{\partial h}{\partial \omega}$. Note that for a given value of $\gamma$ the value of the function tends to decrease as we increase $\phi$. The function also increases as we decrease $\gamma$. 

**Figure A1**

The indifference curves of the marginal productivity of the ratio of late to early investments as a function of $\phi$ and $\gamma$ when $I_2/I_1 = 2$. 
Consider the CES specification for the technology of human capital formation:

\[ h = \left( \frac{\gamma I_1^\phi + (1 - \gamma) I_2^\phi}{\phi} \right)^{\frac{1}{\phi}} \]

Define \( \omega = \frac{I_2}{I_1} \), the ratio of late to early investments in human capital. From the homogeneity of degree one we can rewrite the technology as:

\[ h = I_1 \left[ \gamma + (1 - \gamma) \omega^\phi \right]^{\frac{1}{\phi}}. \]

The marginal product of the ratio of late to early investment, \( \omega \), holding early investment constant, is

\[ \frac{\partial h}{\partial \omega} = (1 - \gamma) I_1 \left[ \gamma + (1 - \gamma) \omega^\phi \right]^{\frac{1-\phi}{\phi}} \omega^{\phi-1} \]

This figure displays the indifference curves of \( \frac{\partial h}{\partial \omega} \) when \( \omega = 0.5 \). Each indifference curve shows the corresponding level of \( \frac{\partial h}{\partial \omega} \). Note that for a given value of \( \gamma \) the value of the function tends to decrease as we increase \( \phi \). However, the function may not be monotonic with respect to \( \gamma \).
Table 7. The Ratio of Optimal Early and Late Investments $\frac{I_1}{I_2}$ Under Different Assumptions About the Skill Formation Technology

<table>
<thead>
<tr>
<th>Assumption</th>
<th>Low Self-Productivity: $\gamma &lt; \frac{(1+r)}{(2+r)}$</th>
<th>High Self-Productivity: $\gamma &gt; \frac{(1+r)}{(2+r)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>High Degree of Complementarity: $\phi &lt; 0$</td>
<td>$\frac{I_1}{I_2} \to 1$ as $\phi \to -\infty$</td>
<td>$\frac{I_1}{I_2} \to 1$ as $\phi \to -\infty$</td>
</tr>
<tr>
<td>Low Degree of Complementarity: $0 \leq \phi \leq 1$</td>
<td>$\frac{I_1}{I_2} \to 0$ as $\phi \to 1$</td>
<td>$\frac{I_1}{I_2} \to \infty$ as $\phi \to 1$</td>
</tr>
</tbody>
</table>

Note: This table summarizes the behavior of the ratio of optimal early to late investments according to four cases: $I_1$ and $I_2$ have high complementarity, but self-productivity is low; $I_1$ and $I_2$ have both high complementarity and self-productivity; $I_1$ and $I_2$ have low complementarity and self-productivity; and $I_1$ and $I_2$ have low complementarity, but high self-productivity. When $I_1$ and $I_2$ exhibit high complementary, complementarity dominates and is a force towards equal distribution of investments between early and late periods. Consequently, self-productivity plays a limited role in determining the ratio $\frac{I_1}{I_2}$ (row 1). On the other hand, when $I_1$ and $I_2$ exhibit a low degree of complementarity, self-productivity tends to concentrate investments in the late period if self-productivity is low, but in the early period if it is high (row 2).
B Comparing the Technology of Skill Formation to the Ben-Porath Model

B.1 The General Technology of Skill Formation

Let $\theta_t$ be a $L \times 1$ vector of skills or abilities at stage $t$. Included are pure cognitive abilities (e.g. IQ) as well as noncognitive abilities (time preference, self control, patience, judgment). The notation is sufficiently flexible to include acquired skills like general education or a specific skill. Agents start out life with vector $\theta_1$ of skills (abilities). The $\theta_1$ are produced by genes and in utero environments which are known to affect child outcomes (see, e.g., the essays in Keating and Hertzman, 1999).

Let $I_t$ be a $K \times 1$ vector of investments at stage $t$. These include all inputs invested in the child including parental and social inputs. The technology of skill formation can be written as

$$\theta_{t+1} = f_t(\theta_t, I_t)$$

where $f_t$ is a stage-$t$ function mapping skill (ability) levels and investment at stage $t$ into skill (ability) levels at the end of the period. For simplicity we assume that $f_t$ is twice continuously differentiable in its arguments. Its domain of definition is the same for all inputs. The inputs may be different at different stages of the life cycle, so the inputs in $I_t$ may be different from the inputs at period $\tau$ different from $t$.

Universal self-productivity at stage $t$ is defined as

$$\frac{\partial \theta_{t+1}}{\partial \theta_t} = \frac{\partial f_t}{\partial \theta_t} > 0.$$  

In the general case this is a $L \times L$ matrix. More generally, some components of this matrix may be zero at all stages while other components may always be positive. In principle, some skills could have negative effects in some periods. At some stages, some components may be zero while at other stages they may be positive.

Universal direct complementarity at stage $t$ is defined by the $L \times K$ matrix:

$$\frac{\partial^2 \theta_{t+1}}{\partial \theta_t \partial I_t} > 0.$$  

Higher levels of $\theta_t$ raise the productivity of $I_t$. Alternatively, higher levels of
$I_t$ raise the productivity of $\theta_t$. Again, in the general case, some components at some or all stages may have zero effects, and some may have negative effects. They can switch signs across stages.

This notation is sufficiently general to allow for the possibility that some components of skill are produced only at certain critical periods. Period $t$ is critical for skill (ability) $j$ if

$$\frac{\partial \theta_{t+1,j}}{\partial I_t} \neq 0,$$

for some levels of $\theta_t, I_t = i_t$, but

$$\frac{\partial \theta_{t+k+1,j}}{\partial I_{t+k}} = 0, \quad k > 0,$$

for all levels of $\theta_t, I_t = i_t$.

Sensitive periods might be defined as those periods where, at the same level of input $\theta_t, I_t$, the $\frac{\partial \theta_{t+1,j}}{\partial I_t}$ are high. More formally, letting $\theta_t = 0, I_t = i, t$ is a sensitive period for skill (or ability) $j$ if

$$\frac{\partial \theta_{t+k+1,j}}{\partial I_{t+k}} \bigg|_{\theta_{t+k}=0, I_{t+k}=i} < \frac{\partial \theta_{t+1,j}}{\partial I_t} \bigg|_{\theta_t=0, I_t=i}, \quad \text{for } k \neq 0.$$

Clearly there may be multiple sensitive periods, and there may be sensitivity with respect to one input that is not true of other inputs.

An alternative definition of critical and sensitive periods works with a version of the technology that solves out $\theta_{t+1,j}$ as a function of lagged investments and initial conditions $\theta_1 = \theta_1$: $\theta_{t+1,j} = M_{t,j}(I_t, I_{t-1}, \ldots, I_1, \theta_1), \quad j = 1, \ldots, J$.

Stage $t^*$ is a critical period for $\theta_{t+1,j}$ if investments are productive at $t^*$ but not at any other stage $k \neq t^*$. Formally,

$$\frac{\partial \theta_{t+1,j}}{\partial I_k} \bigg|_{\theta_{t+k}=0, I_{t+k}=i} \equiv 0, \quad k \neq t^*, \quad j = 1, \ldots, J,$$

for all $\theta_1, I_1, \ldots, I_t$, but

$$\frac{\partial \theta_{t+1,j}}{\partial I_{t^*}} \bigg|_{\theta_{t}+1,j} = \frac{\partial M_{t,j}(I_t, I_{t-1}, \ldots, I_1, \theta_0)}{\partial I_{t^*}} > 0, \quad j = 1, \ldots, J,$$

for some $\theta_1, I_1, \ldots, I_t$. 
Stage $t^*$ is a sensitive period for $\theta_{t+1,j}$ if at the same level of inputs, investment is more productive at stage $t^*$ than at stage $t$. Formally, $t^*$ is a sensitive period for $\theta_{t+1,j}$ if for $k \neq t^*$,

$$\frac{\partial \theta_{t+1,j}}{\partial I_k} \bigg|_{\theta_1=\theta_1, I_t=I_t, k=1,...,t, k \neq t^*} \leq \frac{\partial \theta_{t+1,j}}{\partial I_{t^*}} \bigg|_{\theta_1=\theta_1, I_{t^*}=I_{t^*}, k=1,...,t} .$$

The inequality is strict for at least one period $k = 1, \ldots, t, k \neq t^*$.

This definition of critical periods agrees with the previous one. Our second definition of sensitive periods may not agree with the previous one, which is defined only in terms of the effect of investment on the next period’s output. The second definition fixes the period at which output is measured and examines the marginal productivity of inputs in producing the output. It allows for feedback effects of the investment in $j$ on output beyond $j$ through self-productivity in a way that the first definition does not.

At each stage $t$, agents can perform certain tasks. The level of performance in task $l$ at stage $t$ is $T_{l,t} = T_{l,t} (\theta_t)$. For some tasks, and some stages, components of $\theta_t$ may be substitutes or complements. Thus we can, in principle, distinguish complementarity or substitution in skills (abilities) in stage $t$ in task performance from complementarity or substitution in skill production. Agents deficient in some skills may specialize in some tasks. This is an alternative form of remediation compared to remediation through skill investment (see CHLM).

### B.2 Relationship with the Ben-Porath (1967) Model

The conventional formulation of the technology of skill formation is due to Ben-Porath, who applied the concept of a production function to the formation of adult skills. Our analysis has many distinctive features which we elaborate after first reviewing his analysis. Let $\theta_t$ be scalar human capital. This corresponds to a model with one skill (general human capital). His model postulates that human capital at time $t + 1$ depends on human capital at $t$, invariant ability (denoted $\kappa$), and investment at $t$, $I_t$. $I_t$ may be a vector. The same type of investments are made at each stage. Skill is measured in the same units over time. His specification of the investment technology is

$$\theta_{t+1} = f (I_t, \theta_t, \kappa)$$
where $f$ is concave in $I_t$. The technology is specialized further to allow for depreciation of scalar human capital at rate $\rho$. Thus we obtain

$$\theta_{t+1} = g(I_t, \theta_t, \kappa) + (1 - \rho) \theta_t.$$ 

When $\rho = 0$, there is no depreciation. “$\theta_t$” is carried over (not fully depreciated) as long as $\rho < 1$.

Self-productivity in his model arises when $$\frac{\partial \theta_{t+1}}{\partial \theta_t} = \frac{\partial g(k, \theta_t, I_t)}{\partial \theta_t} + (1 - \rho) > 0.$$ 
This comes from two sources: a carry over effect, $(1 - \rho) > 0$, arising from the human capital that is not depreciated, and the effect of $\theta_t$ on gross investment $\left(\frac{\partial g(k, \theta_t, I_t)}{\partial \theta_t} > 0\right)$. If $g(I_t, \theta_t, \kappa) = \phi_1(\theta_t, \kappa) + \phi_2(I_t, \kappa)$, there is no essential distinction between $(1 - \rho) \theta_t$ and $g(I_t, \theta_t, \kappa)$ as sources of self-productivity if we allow $\rho$ to depend on $\kappa (\rho (\kappa))$.

Complementarity of all inputs is defined as

$$\frac{\partial^2 g(I_t, \theta_t, \kappa)}{\partial \theta_t \partial I_t'} > 0.$$ 

In a more general case, some components of this vector may be negative or zero. In the case of universal complementarity, the stock of $\theta_t$ raises the marginal productivity of $I_t$. Direct complementarity and self-productivity, singly and together, show why skill begets skill. Our model generalizes the Ben-Porath model by (a) allowing for different skill formation technologies at different stages to capture the notion of critical and sensitive periods; (b) allowing qualitatively different investments at different stages; (c) allowing for both skill and ability formation and (d) considering the case of vector skills and abilities.

His model features the opportunity cost of time as an essential ingredient. His “neutrality assumption”, $(\theta_{t+1} = f(I_t, \theta_t, \kappa))$, guarantees that productivity in the market (opportunity costs) increases at the same rate as productivity of human capital in self production. For an analysis of parental investment in young children, child time and its opportunity costs are not relevant. Thus, his neutrality assumption is not relevant. In the original Ben-Porath paper, a Cobb-Douglas technology is used. We allow for more general substitution possibilities among investments.
C An OLG/Complete Markets Model

Consider the following formulation of the problem in the text in a complete markets framework. An individual lives for $2T$ years. The first $T$ years the individual is a child of an adult parent. From age $T + 1$ to $2T$ the individual lives as an adult and is the parent of a child. The individual dies at the end of the period in which he is $2T$ years-old, just before his child’s child is born. This is an overlapping generations model in which at every calendar year there are an equal and large number of individuals of every age $t \in \{1, 2, ..., 2T\}$.

A household consists of an adult parent, born in generation $g$, and his child, will be the next generation of this dynasty, generation $g + 1$. In what follows, we will use the subscript $g$ to denote the state and control variables of the parent, and $g + 1$ to denote those of the child.\(^6\) Furthermore, note that when the child is $t$ years-old, the parent is $T + t$ years-old. It suffices to keep track of the age of the child.

Children are assumed to make no decisions. The parents invest in their children because of altruism. We first consider the case in which all parents have the same preferences and supply labor inelastically. Let $I_{g,t}$ denote parental investments in child skill when the child is $t$ years-old, where $t = 1, 2, \ldots, T$. The output of the investment process is a skill vector.

We now describe how skills evolve over generations. Assume that each agent is born with initial conditions $\theta_{g+1,1}$. Let $h_{g}$ denote the parental characteristics (e.g., their IQ, education, etc.). Let $h_{g+1}$ denote the level of skills as the child starts adulthood.\(^7\) The technology of skill formation is

$$h_{g+1} = m_2\left(\theta_{g+1,1}, h_{g}, I_{g,1}, I_{g,2}\right).$$  \hspace{1cm} (C-1)

At the beginning of adulthood, the parents of generation $g$ draw two random variables: the initial level of skill of the child, $\theta_{g+1,1}$ and a permanent shock $\varepsilon_g$ to their human capital endowment. We use $p\left(\theta_{g+1,1}, \varepsilon_g\right)$ to denote the joint density of these random variables. Let $q\left(\theta_{g+1,1}, \varepsilon_g\right)$ denote the price of a claim that delivers one unit of consumption good if the initial skill level is $\theta_{g+1,1}$ and the permanent shock is $\varepsilon_g$ and zero otherwise. Upon reaching adulthood, the parents receive a bequest that contains

\(^6\)Note that if the parent is born in year $y$ the child will be born in year $y + T$. We define generations in terms not of year of birth but order of birth within a dynasty.

\(^7\)In the text, we use $h'$ to denote this value. Here we use a generationally consistent notation.
of such claims. For every generation $g$, the state variables for the parent are described by the vector \( \psi_g = (\varepsilon_g, \theta_{g+1,1}, h_g, b_{g}(\theta_{g+1,1}, \varepsilon_g)) \).

Let \( c_{g,1} \) and \( c_{g,2} \) denote the consumption of the household in the first and second period of the lifecycle of the child. The parents decide how to allocate the resources among consumption and investments at different periods as well as bequests in claims \( b_{g+1}(\theta, \varepsilon) \) which may be positive or negative. Assuming that human capital (parental and child) is scalar, the budget constraint is

\[
c_{g,1} + \frac{c_{g,2}}{(1 + r)} + \frac{\int b_{g+1}(\theta, \varepsilon) q(\theta, \varepsilon) d\theta d\varepsilon}{(1 + r)^2} = \frac{wh_g}{(1 + r)} + \frac{wh_g}{(1 + r)} + b_{g+1}(\theta_{g+1,1}, \varepsilon_g).
\]

Let \( \beta \) denote the utility discount factor and \( \delta \) denote the parental altruism toward the child. Let \( u(.) \) denote the utility function. The recursive formulation of the problem of the parent is

\[
V(\varepsilon_g, \theta_{g+1,1}, h_g, b_g(\theta_{g+1,1}, \varepsilon_g)) = \max\left\{ u(c_{g,1}) + \beta u(c_{g,2}) + \beta^2 \delta \mathbb{E}[V(\varepsilon_{g+1}, \theta_{g+2,1}, h_{g+1}, b_g(\theta_{g+2,1}, \varepsilon_{g+1}))]\right\}.
\]  

The problem of the parent is to maximize (C-3) subject to the budget constraint (C-2) and the technology of skill formation (C-1). The first-order conditions for investments are:

\[
\delta \beta^2 \mathbb{E}\left( \frac{\partial V}{\partial h_{g+1}} \right) \frac{\partial h_{g+1}}{\partial I_{g,1}} = u'(c_{g,1}) \quad \text{(C-4)}
\]

\[
\delta \beta^2 \mathbb{E}\left( \frac{\partial V}{\partial h_{g+1}} \right) \frac{\partial h_{g+1}}{\partial I_{g,2}} = \frac{u'(c_{g,1})}{1 + r} \quad \text{(C-5)}
\]

The levels of investments do not depend on parental resources. Bad draws for the productivity shock \( \varepsilon_g \) are protected by the financial markets through a larger bequest \( b(\theta_{g+1,1}, \varepsilon_g) \). The early and late investments are determined so that the marginal cost of an extra unit of investment is equal to the marginal expected payoff of that same unit:

\[
\delta \beta^2 \sum_{\tau=1}^{\infty} \frac{1}{(1 + r)^{2\tau}} \mathbb{E}\left[ \left(1 + \frac{1}{1 + r}\right) w_{\varepsilon_{g+\tau}} \frac{\partial h_{g+\tau}}{\partial h_{g+1}} \frac{\partial h_{g+1}}{\partial I_{g,1}} \right] = 1 \quad \text{(C-6)}
\]
Derivations for the Dynamic Complete Market Case

To see how to obtain equations (C-6) and (C-7), consider the first-order condition for $b_{g+1}(\theta_{g+2,1}, \varepsilon_{g+1})$:

$$q(\theta_{g+2,1}, \varepsilon_{g+1}) \frac{u'(c_{g,1})}{(1+r)^2} = \beta^2 \frac{\partial V}{\partial b_{g+1}} \left( \theta_{g+2,1}, \varepsilon_{g+1} \right) p(\theta_{g+2,1}, \varepsilon_{g+1}).$$

(C-8)

Suppose that the price of the claim is actuarially fair, so that $q(\theta_{g+2,1}, \varepsilon_{g+1}) = p(\theta_{g+2,1}, \varepsilon_{g+1})$. If we use the Benveniste-Scheinkman Theorem (Benveniste and Scheinkman, 1979) to compute $\frac{\partial V}{\partial h_{g+1}}(\theta_{g+2,1}, \varepsilon_{g+1})$ it follows that

$$\frac{u'(c_{g,1})}{(1+r)^2} = \beta^2 u'(c_{g+1,1}).$$

(C-9)

Let $A_g = w + \frac{w}{1+r}$. We apply the Benveniste-Scheinkman Theorem to compute $\frac{\partial V}{\partial h_g}$:

$$\frac{\partial V}{\partial h_g} = u'(c_{g,1}) A_g \varepsilon_g + \beta^2 E \left[ \frac{\partial V}{\partial h_{g+1}} \frac{\partial h_{g+1}}{\partial h_g} \right].$$

(C-10)

A recursive application of the Benveniste-Scheinkman Theorem on $\frac{\partial V}{\partial h_{g+1}}$ shows that

$$\frac{\partial V}{\partial h_{g+1}} = u'(c_{g+1,1}) A_{g+1} \varepsilon_{g+1} + \beta^2 E \left[ \frac{\partial V}{\partial h_{g+2}} \frac{\partial h_{g+2}}{\partial h_{g+1}} \right].$$

(C-11)

Replacing (C-10) in (C-9) we obtain

$$\frac{\partial V}{\partial h_g} = u'(c_{g,1}) A_g \varepsilon_g + \beta^2 E \left[ u'(c_{g+1,1}) A_{g+1} \varepsilon_{g+1} \frac{\partial h_{g+1}}{\partial h_g} \right] + \beta^4 E \left[ \frac{\partial V}{\partial h_{g+2}} \frac{\partial h_{g+2}}{\partial h_g} \right].$$
Continuing with the recursion we conclude that

\[ \frac{\partial V}{\partial h_g} = \sum_{\tau=0}^{\infty} (\delta^2)^\tau E \left[ u' \left( c_{g+\tau,1} \right) A_{g+\tau} \varepsilon_{g+\tau} \frac{\partial h_{g+\tau}}{\partial h_g} \right]. \]

Now, use (C-8) to obtain

\[ \frac{\partial V}{\partial h_g} = u' \left( c_{g,1} \right) \sum_{\tau=0}^{\infty} \frac{1}{(1+\tau)^2} E \left[ A_{g+\tau} \varepsilon_{g+\tau} \frac{\partial h_{g+\tau}}{\partial h_g} \right]. \]  \hspace{0.5cm} \text{(C-11)}

By replacing (C-11) into (C-4) and (C-5) we obtain (C-6) and (C-7).
D Program Definitions, Tables and Figures

Descriptions of Intervention Programs Discussed in the Text

**Head Start.** Head Start is a national program targeted to low-income preschool aged children (ages 3–5) that promotes school readiness by enhancing their social and cognitive development through the provision of educational, health, nutritional, social and other services to enrolled children and families. There is a new program, Early Head Start, that begins at age 1.

**Perry Preschool Program.** The Perry preschool experiment was an intensive family enhancement preschool program administered to randomly selected disadvantaged black children enrolled in the program over five different waves between 1962 and 1967. Children were enrolled $2\frac{1}{2}$ hours per day, 5 days a week, during the school year and there were weekly 1$\frac{1}{2}$-hour home visits. They were treated for 2 years, ages 3 and 4. A control group provides researchers with an appropriate benchmark to evaluate the effects of the preschool program.

**The Abecedarian Project.** The Abecedarian Project recruited children born between 1972 and 1977 whose families scored high on a “High Risk” index. It enrolls and enriches the family environments of disadvantaged children beginning a few months after birth and continuing until age 5. At age 5—just as they were about to enter kindergarten—all of the children were reassigned to either a school age intervention through age 8 or to a control group. The Abecedarian program was more intensive than the Perry program. Its preschool program was a year-round, full-day intervention.

**Chicago Parent-Child Center.** The CPC was started in 1967, in selected public schools serving impoverished neighborhoods of Chicago. Using federal funds, the center provided half-day preschool program for disadvantaged 3- and 4-year-olds during the 9 months that they were in school. In 1978, state funding became available, and the program was extended through third grade and included full-day kindergarten.
Tables

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D2 Test Scores and the Timing of Income: White Males, CNLSY/1979

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D0 Trend in Mean Cognitive Score by Maternal Education
D00 Children of NLSY: Average Standardized Score, Peabody Picture Vocabulary Test by Permanent Income Quartile
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D9c  Perry Preschool Program: economic effects at age 27, by treatment group
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D10a College participation of HS graduates and GED holders: white males
D10b College participation by race: dependent high school graduates and GED holders (males, 18–24)
Table D1 Economic Benefits And Costs

<table>
<thead>
<tr>
<th></th>
<th>Perry</th>
<th>Chicago CPC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Child Care</td>
<td>986</td>
<td>1,916</td>
</tr>
<tr>
<td>Earnings</td>
<td>40,537</td>
<td>32,099</td>
</tr>
<tr>
<td>K-12</td>
<td>9,184</td>
<td>5,634</td>
</tr>
<tr>
<td>College/Adult</td>
<td>-782</td>
<td>-644</td>
</tr>
<tr>
<td>Crime</td>
<td>94,065</td>
<td>15,329</td>
</tr>
<tr>
<td>Welfare</td>
<td>355</td>
<td>546</td>
</tr>
<tr>
<td>FG Earnings</td>
<td>6,181</td>
<td>4,894</td>
</tr>
<tr>
<td>Abuse/Neglect</td>
<td>0</td>
<td>344</td>
</tr>
<tr>
<td><strong>Total Benefits</strong></td>
<td>150,525</td>
<td>60,117</td>
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<tr>
<td><strong>Total Costs</strong></td>
<td>16,514</td>
<td>7,738</td>
</tr>
<tr>
<td><strong>Net Present Value</strong></td>
<td>134,011</td>
<td>52,380</td>
</tr>
<tr>
<td><strong>Benefits-To-Costs Ratio</strong></td>
<td>9.11</td>
<td>7.77</td>
</tr>
</tbody>
</table>

Notes: All values discounted at 3% and are in $2004. Numbers differ slightly from earlier estimates because FG Earnings for Perry and Chicago were estimated using the ratio of FG Earnings Effect to Earnings Effect (about 15%) that was found in Abecedarian.

<table>
<thead>
<tr>
<th>Ability Factor at ages 12-13(^a)</th>
<th>Coefficient</th>
<th>t-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Family Permanent Income 0-14(^b)</td>
<td>0.1899</td>
<td>4.1300</td>
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<tr>
<td>Family Permanent Income 0-4(^c)</td>
<td>-0.1224</td>
<td>-1.5800</td>
</tr>
<tr>
<td>Family Permanent Income 5-9(^d)</td>
<td>-0.0527</td>
<td>-0.6400</td>
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<tr>
<td>Family Permanent Income 10-14(^e)</td>
<td>0.0346</td>
<td>0.5900</td>
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<table>
<thead>
<tr>
<th>Ability Factor at ages 12-13(^a)</th>
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<th>t-statistic</th>
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</thead>
<tbody>
<tr>
<td>Mother's Ability(^f)</td>
<td>0.2729</td>
<td>9.5700</td>
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</table>

<table>
<thead>
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<th>Ability Factor at ages 12-13(^a)</th>
<th>Coefficient</th>
<th>t-statistic</th>
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</thead>
<tbody>
<tr>
<td>Mother's Age at Test Date</td>
<td>0.0070</td>
<td>1.1500</td>
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<thead>
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<th>Ability Factor at ages 12-13(^a)</th>
<th>Coefficient</th>
<th>t-statistic</th>
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<tbody>
<tr>
<td>Constant</td>
<td>-0.9315</td>
<td>-3.8600</td>
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</table>

<table>
<thead>
<tr>
<th>Ability Factor at ages 12-13(^a)</th>
<th>Coefficient</th>
<th>t-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Observations</td>
<td>883</td>
<td>855</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Ability Factor at ages 12-13(^a)</th>
<th>Coefficient</th>
<th>t-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>R(^2)</td>
<td>0.1748</td>
<td>0.1718</td>
</tr>
</tbody>
</table>

\(^a\)The Ability Factor at ages 12-13 is obtained in the following way. First, we standardize the scores in PIAT Math and PIAT Reading Recognition so that at each age each score has mean zero and variance one. Then, we factor analyze the scores at each age and extract one factor. The factor values at age 12 and age 13 are combined to form only one factor. Let f\(_{12}\), f\(_{13}\) denote the factors at age 12 and 13, respectively. Let d\(_{12}\) take the value one if the factor at age 12 is nonmissing. Let d\(_{13}\) take the value one if the factor at age 13 is nonmissing. Then we construct the factor f as f = d\(_{12}\)*f\(_{12}\) + (1-d\(_{12}\))*d\(_{13}\)*f\(_{13}\).

\(^b\)Family Permanent Income from age 0 to age 14 of the child. It is the average (inflation-adjusted) family income from age 0 to age 14 of the child.

\(^c\)Family Permanent Income from age 0 to age 4 of the child. It is the average (inflation-adjusted) family income from age 0 to age 4 of the child.

\(^d\)Family Permanent Income from age 5 to age 9 of the child. It is the average (inflation-adjusted) family income from age 5 to age 9 of the child.

\(^e\)Family Permanent Income from age 10 to age 14 of the child. It is the average (inflation-adjusted) family income from age 10 to age 14 of the child.

\(^f\)AFQT of the Mother (NLSY/79).
This figure shows the average standardized score in the PIAT Math test from ages 5 to 14 by quartile of family permanent income. The sample consists of all Children of NLSY/79. Family permanent income is the mean family income from age 0 to age 18 of the child. At each age, we standardize the PIAT math score so it has mean zero and variance one. That is, let $m_{i,t}$ denote the score of child $i$ at age $t$. Let $\mu_t, \sigma_t^2$ denote the mean and variance of the PIAT-Math score at age $t$. We construct the variable $z_{i,t}$ as:

$$z_{i,t} = \frac{m_{i,t} - \mu_t}{\sigma_t}$$

We then proceed by calculating the mean $z_{i,t}$ by quartile of family income. Let $1(q_i = Q_j)$ denote the function that takes the value one if the family permanent income of child $i$ is in quartile $Q_j$ and zero otherwise. Let $\bar{z}_{j,t}$ denote the mean standardized score at age $t$ of the children whose permanent income is in quartile $Q_j$:

$$\bar{z}_{j,t} = \frac{\sum_i z_{i,t} 1(q_i = Q_j)}{\sum_i 1(q_i = Q_j)}$$
The dramatic results on the importance of the early years in creating differences among children arise if “Bayley scores” are used as a measure of cognition at age 1. As Michael Lewis and Harry McGurk (1972) point out, this is illegitimate since the Bayley score tests other aspects of child development in addition to cognition.
Figure D00
Children of NLSY
Average Standardized Score
Peabody Picture Vocabulary Test by Permanent Income Quartile

Source: Full Sample of Children of the National Longitudinal Survey of Youth.
Please see our website for a full explanation of this figure.
Figure D1a
Children of NLSY

Average percentile rank on PIAT Math score, by income quartile*

*Income quartiles are computed from average family income between the ages of 6 and 10.
Figure D1b
Children of NLSY

Adjusted average PIAT Math score percentiles by income quartile*

* Adjusted by maternal education, maternal AFQT (corrected for the effect of schooling) and broken home at each age
Figure D2a
Children of NLSY
Average percentile rank on anti-social behavior score, by income quartile*
Figure D2b
Children of NLSY

Adjusted average anti-social behavior score percentile by income quartile*

* Adjusted by maternal education, maternal AFQT (corrected for the effect of schooling) and broken home at each age
Figure D3a
Average percentile rank on anti-social behavior score, by race
Figure D3b
Adjusted average anti-social behavior score percentile by race*

* Adjusted by maternal education, maternal AFQT (corrected for the effect of schooling) and broken home at each age
Figure D4a

Early Childhood Longitudinal Study (ECLS)

(a) Reading

Source: Raudenbush (2006)
Figure D4b
Mean trajectories, high and low poverty schools (ECLS)

(b) Math

Source: Raudenbush (2006)
Average trajectories, Grades 1-3, high and low poverty schools (Sustaining Effects Study)

(a) Reading

Source: Raudenbush (2006)
Average trajectories, Grades 1-3, high and low poverty schools (Sustaining Effects Study)

(b) Math

Source: Raudenbush (2006)
Figure D6a

Average achievement trajectories, Grades 8-12 (NELS 88).

(a) Science

Source: Raudenbush (2006)
Figure D6b

Average achievement trajectories, Grades 8-12 (NELS 88).

(b) Math

Source: Raudenbush (2006)
Growth as a function of student social background: ECLS

Figure D7a

(a) Reading

Source: Raudenbush (2006)
Figure D7b

Growth as a function of student social background: ECLS

(b) Math

Source: Raudenbush (2006)
Figure D8a

Growth as a Function of School Poverty for Poor Children: Sustaining Effects Data

(a) Reading

Source: Raudenbush (2006)
Figure D8b

Growth as a Function of School Poverty for Poor Children: Sustaining Effects Data

(b) Math

Source: Raudenbush (2006)
Figure D9a
Perry Preschool Program: IQ, by Age and Treatment Group

Source: Perry Preschool Program. IQ measured on the Stanford–Binet Intelligence Scale (Terman & Merrill, 1960). Test was administered at program entry and each of the ages indicated.
On Time Grad. from HS

High Achievement at Age 14*

Special Education

Notes: *High achievement defined as performance at or above the lowest 10th percentile on the California Achievement Test (1970).
Figure D9c
Perry Preschool Program: Economic Effects at Age 27, by Treatment Group

- **Earn +$2,000 Monthly**: Treatment 29%, Control 7%
- **Own Home**: Treatment 36%, Control 13%
- **Never on Welfare as Adult**: Treatment 29%, Control 14%

Source: Barnett (2004). *Updated through Age 40 using recent Perry Preschool Program data, derived from self-report and all available state records.*
Figure D9d
Perry Preschool Program: Arrests per Person before Age 40, by Treatment Group

Source: Perry Preschool Program. Juvenile arrests are defined as arrests prior to age 19.
D10a: College participation of HS graduates and GED holders

Dependent* white males, ages eighteen to twenty-four

<table>
<thead>
<tr>
<th>Year</th>
<th>Family Income Bottom Quartile</th>
<th>Family Income Third Quartile</th>
<th>Family Income Top Half</th>
</tr>
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<tbody>
<tr>
<td>1970</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1971</td>
<td></td>
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Source: Computed from the CPS P-20 School Reports and the October CPS.

*Dependent is living at parental home or supported by parental family while at college.

White Females

High School Graduates of Regular Day School Programs, Public and Private
Percentage of 17 Year Olds

USA, 1971-1999

Source:
(1) the Department of Education National Center for Education Statistics and (2) American Council on Education, General Educational Development Testing Service.
D10b: College participation by race
Dependent high school graduates and GED holders
Males, ages eighteen to twenty-four

Note: Three-year moving averages are shown

Source: Computed from the CPS P-20 School Reports and the October CPS.
*Dependent is living at parental home or supported by parental family while at college.
References


