

# Cowles Commission Structural Models, Causal Effects and Treatment Effects: A Synthesis

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Econometric Policy Evaluation, Lecture I  
Koopmans Memorial Lectures  
Cowles Foundation  
Yale University  
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# Monograph 10

## STATISTICAL INFERENCE IN DYNAMIC ECONOMIC MODELS

by

**COWLES COMMISSION RESEARCH STAFF MEMBERS AND  
GUESTS**

Edited by

**TJALLING C. KOOPMANS**

With Introduction by

**JACOB MARSCHAK**



John Wiley & Sons, Inc., New York

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1950



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# Monograph 14

## STUDIES IN ECONOMETRIC METHOD

by

**COWLES COMMISSION RESEARCH STAFF MEMBERS**

Edited by

WILLIAM C. HOOD AND TJALLING C. KOOPMANS



John Wiley & Sons, Inc., New York  
Chapman & Hall, Limited, London  
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## Cowles Commission motto:

For 20 years, the motto of the Cowles Commission, printed on its monographs and reports, was based on Lord Kelvin's dictum paraphrased as,

*"Science is measurement"*



## Cowles Commission motto:

By 1965 the importance of theory for interpreting evidence had become so apparent that the motto was changed to

*“Theory and measurement”*

## It is fitting

For many years at the University of Chicago, Cowles researchers worked in a building carved with the quotation by Lord Kelvin,

*“When you cannot measure, your knowledge is meager and unsatisfactory.”*

## It is fitting

For many years at the University of Chicago, Cowles researchers worked in a building carved with the quotation by Lord Kelvin,

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My lectures build on these works and these themes.

# Introduction

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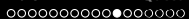
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  - $D = 1$  if it adopts.

# Introduction

- To focus ideas, analyze a prototypical policy evaluation problem.
- Country can adopt a policy (e.g., democracy).
- Choice Indicator:
  - $D = 1$  if it adopts.
  - $D = 0$  if not.



- Two outcomes  $(Y_0(\omega), Y_1(\omega))$ ,  $\omega \in \Omega$



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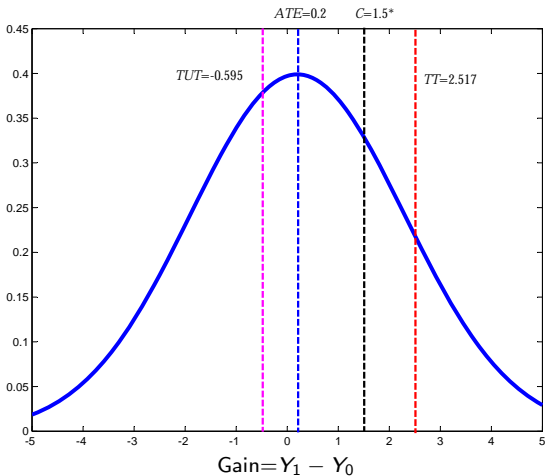


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  - $Y_1(\omega)$  if country adopts
- Causal effect on observed outcomes
- Marshallian *ceteris paribus* causal effect:

$$Y_1(\omega) - Y_0(\omega)$$

# Figure 1: Extended Roy economy for policy adoption

## Distribution of gains and treatment parameters



## Figure 1 Legend

Suppose that a country has to choose whether to implement a policy. Under the policy, the GDP would be  $Y_1$ . Without the policy, the GDP of the country would be  $Y_0$ . For the sake of simplicity, suppose that

$$\begin{aligned}Y_1 &= \mu_1 + U_1 \\Y_0 &= \mu_0 + U_0\end{aligned}$$

where  $U_0$  and  $U_1$  are unobserved components of the aggregate output. The error terms ( $U_0, U_1$ ) are dependent in a general way. Let  $\delta$  denote the additional GDP due to the policy, i.e.  $\delta = \mu_1 - \mu_0$ . We assume  $\delta > 0$ . Let  $C$  denote the cost of implementing the policy. We assume that the cost is a fixed parameter  $C$ .

## Figure 1 Legend

We relax this assumption below. The country's decision can be represented as:

$$D = \begin{cases} 1 & \text{if } Y_1 - Y_0 - C > 0 \\ 0 & \text{if } Y_1 - Y_0 - C \leq 0, \end{cases}$$

so the country decides to implement the policy ( $D = 1$ ) if the net gains coming from it are positive. Therefore, we can define the probability of adopting the policy in terms of the propensity score

$$\Pr(D = 1) = P(Y_1 - Y_0 - C > 0).$$

We assume that  $(U_1, U_0) \sim N(\mathbf{0}, \Sigma)$ ,  $\Sigma = \begin{bmatrix} 1 & -0.5 \\ -0.5 & 1 \end{bmatrix}$ ,  $\mu_0 = 0.67$ ,  $\delta = 0.2$  and  $C = 1.5$ .





- More generally, define outcomes corresponding to state (policy, treatment)  $s$  for an “agent” characterized by  $\omega$  as  $Y(s, \omega)$ ,  $\omega \in \Omega = [0, 1]$ ,  $s \in \mathcal{S}$ , set of possible treatments.



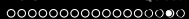
- More generally, define outcomes corresponding to state (policy, treatment)  $s$  for an “agent” characterized by  $\omega$  as  $Y(s, \omega)$ ,  $\omega \in \Omega = [0, 1]$ ,  $s \in \mathcal{S}$ , set of possible treatments.
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- The agent can be any economic agent such as a household, a firm, or a country.
- The  $Y(s, \omega)$  are *ex post* outcomes realized after treatments are chosen.
- Consider uncertainty and related *ex ante* and *ex post* evaluations in the Friday lecture.



- The **individual treatment effect** for agent  $\omega$ .

$$Y(s, \omega) - Y(s', \omega), \quad s \neq s', \quad s, s' \in S, \quad (1.1)$$

**Individual level causal effect.**



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### Individual level causal effect.

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- $R(Y(s, \omega), \omega) > R(Y(s', \omega), \omega)$  if  $s$  is preferred to  $s'$ .



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### Individual level causal effect.

- Comparisons can also be made in terms of utilities  $R(Y(s, \omega))$ .
- $R(Y(s, \omega), \omega) > R(Y(s', \omega), \omega)$  if  $s$  is preferred to  $s'$ .
- The difference in subjective outcomes is  $[R(Y(s, \omega), \omega) - R(Y(s', \omega), \omega)]$ , and is another possible definition of a treatment effect. Holding  $\omega$  fixed holds all features of the person fixed except the treatment assigned,  $s$ .





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- The answer to the question shapes the way we go about policy evaluation analysis.
- A central point in the Cowles research program (Marschak, 1949, 1953).

## P-1

*Evaluating the Impact of Interventions on Outcomes Including Their Impact in Terms of Welfare*

- “Internal validity”: Campbell and Stanley, 1963: looking at a program in place.
- Consider both objective or public outcomes  $Y$  and “subjective” outcomes  $R$ .
- Objective outcomes are intrinsically *ex post* in nature. Subjective outcomes can be *ex ante* or *ex post*.
- *Ex ante* expected pain and suffering may be different from *ex post* pain and suffering. Agents may also have *ex ante* evaluations of the objective outcomes that may differ from their *ex post* evaluations.

## P-2

*Forecasting the Impacts (Constructing Counterfactual States) of Interventions Implemented in one Environment in Other Environments, Including Their Impacts In Terms of Welfare.*

“External validity”: This is the problem of projecting evaluations in one environment to another environment.

## P-3

*Forecasting the Impacts of Interventions (Constructing Counterfactual States Associated with Interventions) Never Historically Experienced to Various Environments, Including Their Impacts in Terms of Welfare.*

- The problem of forecasting the effect of a new policy never tried in any environment.
- All three problems entail identification of counterfactuals.
- But they place different demands on models and the data.

- In answering these questions it is important to separate three tasks.



- In answering these questions it is important to separate three tasks.
- In applied work and in statistical analyses of “causality” these tasks are often confused.

## Three policy evaluation problems

**Table 1: Three distinct tasks arising in the analysis of causal models**

Task	Description	Requirements
1	Defining the Set of Hypotheticals or Counterfactuals	A Scientific Theory
2	Identifying Parameters (Causal or Otherwise) from Hypothetical Population Data	Mathematical Analysis of Point or Set Identification
3	Identifying Parameters from Data	Estimation and Testing Theory

- When is  $Y(s, \omega)$  an adequate description of the outcome of a policy?

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- A policy is equated with an assignment mechanism  $s$ .
- In econometric policy evaluation recognizing agent choices, we need a more general approach.

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- It maps  $\omega \in \Omega$  into  $\mathcal{B}$ , a space of constraints or incentives (e.g., taxes, endowments, eligibility).
- $a : \Omega \rightarrow \mathcal{B}$ .

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- For a given  $b \in \mathcal{B}$ , agents choose a particular treatment.
- $\tau : \Omega \times \mathcal{A} \times \mathcal{B} \rightarrow \mathcal{S}, \tau \in \mathcal{T}$ .
- A policy is a pair  $p = (a, \tau)$ .

- In the general case, outcomes depend on  $\omega, s, a, b, \tau$

$$Y(\omega, s, a, b, \tau)$$





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- When can we write:

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- When can we ignore the mechanism  $a \in \mathcal{A}$  and the treatment assignment rule  $\tau \in \mathcal{T}$  in studying outcomes?
- Need invariance postulates

- Policy invariance for objective outcomes:

## PI-1

*For any two constraint assignment mechanisms  $a, a' \in \mathcal{A}$  and incentives  $b, b' \in \mathcal{B}$ , with  $a(\omega) = b$  and  $a'(\omega) = b'$ , and for all  $\omega \in \Omega$ ,  $Y(s, \omega, a, b, \tau) = Y(s, \omega, a', b', \tau)$ , for all  $s \in \mathcal{S}_{\tau(a,b)}(\omega) \cap \mathcal{S}_{\tau(a',b')}(\omega)$  for assignment rule  $\tau$  where  $\mathcal{S}_{\tau(a,b)}(\omega)$  is the image set for  $\tau(a, b)$ . For simplicity we assume  $\mathcal{S}_{\tau(a,b)}(\omega) = \mathcal{S}_{\tau(a,b)}$  for all  $\omega \in \Omega$ .*

- Rules out effects of the constraint assignment mechanism and incentive schedules on realized outcomes.

## PI-2

*For each constraint assignment  $a \in \mathcal{A}$  and  $b \in \mathcal{B}$  and all  $\omega \in \Omega$ ,  $Y(s, \omega, a, b, \tau) = Y(s, \omega, a, b, \tau')$  for all  $\tau$  and  $\tau' \in \mathcal{T}$  with  $s \in \mathcal{S}_{\tau'(a,b)} \cap \mathcal{S}_{\tau(a,b)}$ , where  $\mathcal{S}_{\tau(a,b)}$  is the image set of  $\tau$  with assignment mechanism  $a$  and incentive  $b$ .*

- For simplicity, we assume  $\mathcal{S}_{\tau(a,b)}(\omega) = \mathcal{S}_{\tau'(a,b)}$ ,  $\forall \omega \in \Omega$ .
- Rules out GE, peer effects, and social interactions.
- (PI-1) and (PI-2) say that it doesn't matter how the agent gets the incentives or what they are (PI-1), or who else gets the treatment or how it is chosen (PI-2).

- Given (PI-1) and (PI-2) we can write the outcome as

$$Y(s, \omega).$$

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- Develop a parallel set of invariance assumptions for utilities  $R$ .

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- Develop a parallel set of invariance assumptions for utilities  $R$ .
- First define

$$\mathcal{A}_b(\omega) = \{a \mid a \subseteq \mathcal{A}, a(\omega) = b\}, \omega \in \Omega.$$



## PI-3

For any two constraint assignment mechanisms  $a, a' \in \mathcal{A}$  and incentives  $b, b' \in \mathcal{B}$  with  $a(\omega) = b$  and  $a'(\omega) = b'$ , and for all  $\omega \in \Omega$ ,  $Y(s, \omega, a, b, \tau) = Y(s, \omega, a', b', \tau)$  for all  $s \in \mathcal{S}_{\tau(a,b)}(\omega) \cap \mathcal{S}_{\tau(a',b')}(\omega)$  for assignment rule  $\tau$ , where  $\mathcal{S}_{\tau(a,b)}(\omega)$  is the image set of  $\tau(a, b)$  and for simplicity we assume that  $\mathcal{S}_{\tau(a,b)}(\omega) = \mathcal{S}_{\tau(a,b)}$  for all  $\omega \in \Omega$ . In addition, for any mechanisms  $a, a' \in \mathcal{A}_b(\omega)$ , producing the same  $b \in \mathcal{B}$  under the same conditions, and for all  $\omega$ ,

$$R(s, \omega, a, b, \tau) = R(s, \omega, a', b, \tau).$$

**PI-4**

For each pair  $(a, b)$  and all  $\omega \in \Omega$ ,

$$Y(s, \omega, a, b, \tau) = Y(s, \omega, a, b, \tau')$$

$$R(s, \omega, a, b, \tau) = R(s, \omega, a, b, \tau')$$

for all  $\tau, \tau' \in \mathcal{T}$  and  $s \in \mathcal{S}_{\tau(a,b)} \cap \mathcal{S}_{\tau'(a,b)}$ .

## How To Construct Counterfactuals?

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- Even a perfectly implemented social experiment does not solve this problem.
- Randomization with full compliance identifies only one component of  $\{Y(s, \omega)\}_{s \in \mathcal{S}}$  for any person.
- In addition, some of the  $s \in \mathcal{S}$  may never be observed.



## The evaluation problem

- For each policy regime, at any point in time we observe person  $\omega$  in some state but not in any of the other states.



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- Let  $D(s, \omega) = 1$  if we observe person  $\omega$  in state  $s$  under policy regime  $p$ .
- Observed objective outcome

$$Y(\omega) = \sum_{s \in \mathcal{S}} D(s, \omega) Y(s, \omega). \quad (2.1)$$



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- The **selection problem** arises because we only observe certain persons in any state.
- We observe  $Y(s, \omega)$  only for persons for whom  $D(s, \omega) = 1$ .
- In general, the outcomes of persons found in  $S = s$  are not representative of what the outcomes of people would be if they were randomly assigned to  $s$ .

- The Roy model (1951): Two possible treatment outcomes ( $S = \{0, 1\}$ ) and a scalar outcome measure and a particular assignment mechanism
$$D(1, \omega) = \mathbf{1}[Y(1, \omega) > Y(0, \omega)]$$
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(reveals  $R(1, \omega) - R(0, \omega) \geq 0$ ).
- The economist's use of choice data distinguishes the econometric approach from the statistical approach.

## How To Construct Counterfactuals?

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- The first avenue, featured in explicitly formulated econometric models and often called “structural econometric analysis”, derives from the Cowles tradition.
- Models  $Y(s, \omega)$  explicitly in terms of its determinants as specified by theory.
- This entails describing the random variables characterizing  $\omega$  and carefully distinguishing what agents know and what the analyst knows.

## How To Construct Counterfactuals?

- This approach also models  $D(s, \omega)$  and the dependence between  $Y(s, \omega)$  and  $D(s, \omega)$  produced from variables common to  $Y(s, \omega)$  and  $D(s, \omega)$ .

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- Specifies a full model and attempts to address problems (P-1)–(P-3).

## How To Construct Counterfactuals?

- A second avenue, pursued in the recent treatment effect literature, redirects attention away from estimating the determinants of  $Y(s, \omega)$  toward estimating some population version of individual “causal effects,” without modeling what factors give rise to the outcome or the relationship between the outcomes and the mechanism selecting outcomes.



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- Agent valuations of outcomes are typically ignored.
- The treatment effect literature focuses largely on policy problem (P-1) for the subset of outcomes that is observed.
- Seeks to answer a narrower problem.

- For program (state, treatment)  $j$  compared to program (state, treatment)  $k$ ,

$$ATE(j, k) = E(Y(j, \omega) - Y(k, \omega)).$$

$$TT(j, k) = E(Y(j, \omega) - Y(k, \omega) \mid D(j, \omega) = 1). \quad (2.2)$$

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- These are the traditional parameters for average returns.
- But for economic analysis, marginal returns are more important.

- The distinction between the marginal and average return is a central concept in economics.

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- The **Effect Of Treatment for People at the Margin of Indifference** (EOTM) between  $j$  and  $k$ , given that these are the best two choices available is, with respect to personal preferences, and with respect to choice-specific costs  $C(j, \omega)$ .



$$\text{EOTM}^R(j, k) = \tag{2.3}$$

$$E \left( \begin{array}{l} Y(j, \omega) \\ -Y(k, \omega) \end{array} \middle| \begin{array}{l} R(Y(j, \omega), C(j, \omega), \omega) = R(Y(k, \omega), C(k, \omega), \omega); \\ R(Y(j, \omega), C(j, \omega), \omega) \\ R(Y(k, \omega), C(k, \omega), \omega) \end{array} \right) \geq R(Y(l, \omega), C(l, \omega), \omega) \right),$$

$l \neq j, k.$

- A generalization of this parameter called the **Marginal Treatment Effect**, introduced into the evaluation literature by Björklund and Moffitt (1987).

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- Return to people at the margin of choice.
- Will discuss methods for identifying this return tomorrow.

## Policy relevant treatment effect

- Effect on aggregate outcomes of one policy regime  $p \in \mathcal{P}$  compared to the effect of another policy regime  $p' \in \mathcal{P}$ :

$$\text{PRTE: } E(Y(s_p(\omega), \omega) - Y(s_{p'}(\omega), \omega)), \\ \text{where } p, p' \in \mathcal{P}.$$

$s_p(\omega)$  is treatment allocated under policy  $p$ .

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- Corresponding to this objective outcome is the subjective counterpart:

$$\text{Subjective PRTE: } E(R(s_p(\omega), \omega)) - E(R(s_{p'}(\omega), \omega)), \\ \text{where } p, p' \in \mathcal{P}.$$

- Modern political economy seeks to know the proportion of people who benefit from policy regime  $p$  compared with  $p'$ . Voting Criterion:

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- Option values also interesting: option of having access to a program.
- Uncertainty and regret (covered Friday).

## A generalized Roy model under perfect certainty

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- $Z$ : observed individual variables that affect choices.
- Each state may be characterized by a bundle of attributes, characteristics or qualities  $Q(s)$  that fully characterize the state. If  $Q(s)$  fully describes the state,  $R(s) = R(Q(s))$ .

$$R(s) = \mu_R(s, Z) + v(s, Z, \nu)$$

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$$D(j) = 1 \text{ if } \operatorname{argmax}_{s \in \mathcal{S}} \{R(s)\} = j, \quad (2.4)$$

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- People *self-select* into treatment.



## A two outcome normal example under perfect certainty

- The Roy model (1951) and its extensions (Gronau, 1974; Heckman, 1974; Willis and Rosen, 1979; Heckman, 1990; Carneiro, Hansen, and Heckman, 2003) are at the core of microeconometrics.

$$Y_1 = X\beta_1 + U_1 \quad (2.5a)$$

$$Y_0 = X\beta_0 + U_0, \quad (2.5b)$$

and associated costs (prices) as a function of  $W$

$$C = W\beta_C + U_C. \quad (2.5c)$$



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- Can embed into general equilibrium models (Heckman, Lochner and Taber, 1998; Wolpin and Lee, 2006)

The valuation of “1” relative to “0” is  $R = Y_1 - Y_0 - C$ .  
Substituting from (2.5a)–(2.5c) into the expression for  $R$ :

$$R = X(\beta_1 - \beta_0) - W\beta_C + U_1 - U_0 - U_C,$$

and sectoral choice is indicated by  $D$  where  $D = 1$  if the agent selects 1; = 0 otherwise:

$$D = \mathbf{1} [R > 0].$$



## A two outcome normal example under perfect certainty

- $v = (U_1 - U_0 - U_C), Z = (X, W).$



## A two outcome normal example under perfect certainty

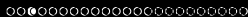
- $v = (U_1 - U_0 - U_C), Z = (X, W).$
- $\gamma = (\beta_1 - \beta_0, -\beta_C).$





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- Thus  $R = Z\gamma + v$ .
- Generalized Roy model:
  - $Z \perp\!\!\!\perp (U_0, U_1, U_C)$  (independence),
  - $(U_0, U_1, U_C) \sim \mathcal{N}(0, \Sigma)$  (normality).

For the Generalized Roy Model, the probability of selecting treatment 1 or “propensity score” is

$$\begin{aligned}\Pr(R > 0 \mid Z = z) &= \Pr(v > -z\gamma) \\ &= \Pr\left(\frac{v}{\sigma_v} > \frac{-z\gamma}{\sigma_v}\right) \\ &= \Phi\left(\frac{z\gamma}{\sigma_v}\right),\end{aligned}$$

where  $\Phi$  is the cumulative distribution function of the standard normal distribution.

- The Average Treatment Effect given  $X = x$  is

$$\begin{aligned} \text{ATE}(x) &= E(Y_1 - Y_0 \mid X = x) \\ &= x(\beta_1 - \beta_0). \end{aligned}$$



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- Treatment on the treated is

$$\begin{aligned} \text{TT}(x, z) &= E(Y_1 - Y_0 \mid Z = z, D = 1) \\ &= x(\beta_1 - \beta_0) + E(U_1 - U_0 \mid v > -Z\gamma, Z = z) \\ &= x(\beta_1 - \beta_0) + E(U_1 - U_0 \mid v > -z\gamma). \end{aligned}$$

- The **local average treatment effect** (LATE) of Imbens and Angrist (1994) is the average gain to program participation for those induced to receive treatment through a change in  $Z [= (X, W)]$  by a component of  $W$  not in  $X$ .



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- This definition is instrument dependent.
- There is a more general approach for defining this parameter (Heckman and Vytlacil, 1999, 2005).



## A two outcome normal example under perfect certainty

- The LATE parameter is the mean return for people with values of  $v \in [\underline{v}, \bar{v}]$ .

$$\text{LATE}(z, z', x)$$

$$= E(Y_1 - Y_0 \mid D(z) = 0, D(z') = 1, X = x)$$

$$= x(\beta_1 - \beta_0)$$

$$+ E(U_1 - U_0 \mid R(z) \leq 0 \cap R(z') > 0, X = x)$$

$$= x(\beta_1 - \beta_0) + E(U_1 - U_0 \mid -z'\gamma < v \leq -z\gamma).$$

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- Instruments  $W$  may not exist yet LATE can still be defined within the economic model as

$$\text{LATE}(x, v \in [\underline{v}, \bar{v}])$$

$$= x(\beta_1 - \beta_0) + E(U_1 - U_0 \mid \underline{v} < v \leq \bar{v}).$$



## A two outcome normal example under perfect certainty

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- Will provide precise conditions for this equivalence in tomorrow's lecture. (See also Heckman and Vytlačil (2005))





- The **marginal treatment effect** (MTE) is defined conditional on  $X$ ,  $Z$ , and  $v = v^*$ :

$$\begin{aligned} & E(Y_1 - Y_0 \mid v = v^*, X = x, Z = z) \\ &= x(\beta_1 - \beta_0) + E(U_1 - U_0 \mid v = v^*). \end{aligned}$$

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- At a special point of evaluation where  $R = 0$  (i.e.  $z\gamma + v = 0$ ), the MTE is a willingness to pay measure that informs us how much an agent at the margin of participation (in the indifference set) would be willing to pay to move from “0” to “1”.

- Under regularity conditions, MTE is a limit form of LATE,

$$\begin{aligned} & \lim_{z\gamma \rightarrow z'\gamma} \text{LATE}(z, z', x) \\ &= x(\beta_1 - \beta_0) + \lim_{z\gamma \rightarrow z'\gamma} E(U_1 - U_0 \mid -z\gamma < v < -z'\gamma) \\ &= x(\beta_1 - \beta_0) + E(U_1 - U_0 \mid v = -z'\gamma) \end{aligned}$$

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- LATE is the average return for persons with  $v \in [-z\gamma, -z'\gamma]$ .

## A two outcome normal example under perfect certainty

- Can work with  $Z\gamma$  or with the propensity score  $P(Z)$  interchangeably assuming  $V$  is absolutely continuous.



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- Can work with  $Z\gamma$  or with the propensity score  $P(Z)$  interchangeably assuming  $V$  is absolutely continuous.
- $\Pr(Z\gamma > V) = \Pr(F_V(Z\gamma) > F_V(V))$ .

## A two outcome normal example under perfect certainty

$$\begin{aligned}
 TT(x, z) &= TT(x, P(z)) \\
 &= x(\beta_1 - \beta_0) \\
 &\quad + \text{Cov}(U_1 - U_0, v) \underbrace{K(P(z))}_{\text{"control function"}} > 0.
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Heckman and Robb (1985)

- Given the model, can build up these and other parameters.

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- But for each of these parameters, we do not need to specify the full model to identify them.
- This is a main insight of the modern treatment effect literature.

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- If we do specify and identify the full model, however, we can solve policy problem (P-2) (the extrapolation problem) using this model evaluated at new values of  $(X, Z)$ .

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- If we do specify and identify the full model, however, we can solve policy problem (P-2) (the extrapolation problem) using this model evaluated at new values of  $(X, Z)$ .
- By construction the  $(U_1, U_0, v)$  are independent of  $(X, Z)$ , and given the functional forms all the mean treatment parameters can be generated for all  $(X, Z)$ .
- By parameterizing the  $\beta_i$  to depend only on measured characteristics, it is possible to forecast the demand for new goods and solve policy problem (P-3).

- Consider the following example.



## A two outcome normal example under perfect certainty

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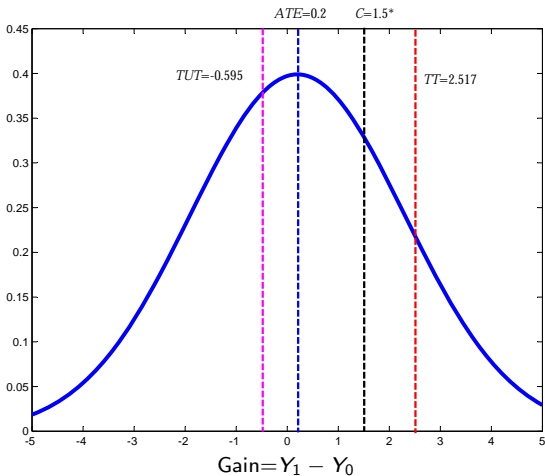
## A two outcome normal example under perfect certainty

- Consider the following example.
- Used throughout these lectures.
- Distribution of gross gains to a country,  $(Y_1 - Y_0)$ , from adopting a policy in a Roy model.

A two outcome normal example under perfect certainty

## Figure 1: Extended Roy economy for policy adoption

### Distribution of gains and treatment parameters





## Figure 1 Legend

Suppose that a country has to choose whether to implement a policy. Under the policy, the GDP would be  $Y_1$ . Without the policy, the GDP of the country would be  $Y_0$ . For the sake of simplicity, suppose that

$$Y_1 = \mu_1 + U_1$$

$$Y_0 = \mu_0 + U_0$$

where  $U_0$  and  $U_1$  are unobserved components of the aggregate output. The error terms ( $U_0, U_1$ ) are dependent in a general way. Let  $\delta$  denote the additional GDP due to the policy, i.e.  $\delta = \mu_1 - \mu_0$ . We assume  $\delta > 0$ . Let  $C$  denote the cost of implementing the policy. We assume that the cost is a fixed parameter  $C$ .

## Figure 1 Legend

We relax this assumption below. The country's decision can be represented as:

$$D = \begin{cases} 1 & \text{if } Y_1 - Y_0 - C > 0 \\ 0 & \text{if } Y_1 - Y_0 - C \leq 0, \end{cases}$$

so the country decides to implement the policy ( $D = 1$ ) if the net gains coming from it are positive. Therefore, we can define the probability of adopting the policy in terms of the propensity score

$$\Pr(D = 1) = P(Y_1 - Y_0 - C > 0).$$

We assume that  $(U_1, U_0) \sim N(\mathbf{0}, \mathbf{\Sigma})$ ,  $\mathbf{\Sigma} = \begin{bmatrix} 1 & -0.5 \\ -0.5 & 1 \end{bmatrix}$ ,  $\mu_0 = 0.67$ ,  $\delta = 0.2$  and  $C = 1.5$ .

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- The average return for the adopting countries is TT (= 2.52).
- Thus the countries adopting the policy are the ones who benefit from it. This is a source of evaluation bias in comparing policy effectiveness in different countries.

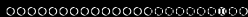


## A two outcome normal example under perfect certainty

Parameter	Definition	Under Assumptions(*)
Marginal Treatment Effect	$E[Y_1 - Y_0 \mid R = 0, P(Z) = p]$	$\delta + \Phi_{U_1 - U_0}^{-1}(1 - p)$
Average Treatment Effect	$E[Y_1 - Y_0 \mid P(Z) = p]$	$\varphi$
Treatment on the Treated	$E[Y_1 - Y_0 \mid R > 0, P(Z) = p]$	$\varphi + \frac{\phi_{U_1 - U_0} \left( \Phi_{U_1 - U_0}^{-1}(1 - p) \right)}{p}$
Treatment on the Untreated	$E[Y_1 - Y_0 \mid R \leq 0, P(Z) = p]$	$\varphi - \frac{\phi_{U_1 - U_0} \left( \Phi_{U_1 - U_0}^{-1}(1 - p) \right)}{1 - p}$

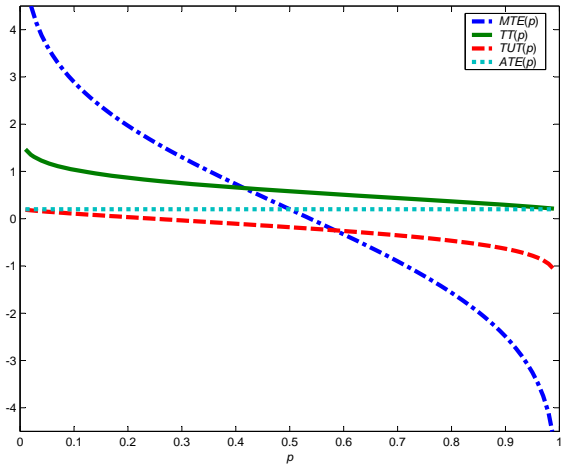
Definitions of treatment parameters for the model given in figure 1.

Figure 2 plots the parameters  $ATE(p)$ ,  $TT(p)$ ,  $MTE(p)$  and  $TUT(p)$  (treatment on the untreated) that underlie the model used to generate figure 1.



A two outcome normal example under perfect certainty

## Figure 2: Extended Roy economy example, treatment parameters as a function of $\Pr(D = 1 \mid Z = z) = p$



Model generated by the parameters from the model at base of Figure 1.



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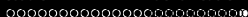
## A two outcome normal example under perfect certainty

- The declining  $MTE(p)$  is the prototypical pattern of diminishing returns that accompanies a policy expansion.
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- As costs  $C$  fall, more countries are drawn in to adopt the policy, the return falls.



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- But do we need to?
- We consider this question but first consider a version of the analysis that allows for uncertainty.

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- In environments of uncertainty, agent choices are made in terms of *ex ante* calculations.
- Yet the treatment effect literature largely reports *ex post* returns.

As Hicks (1946, p. 179) puts it,

*“Ex post calculations of capital accumulation have their place in economic and statistical history; they are useful measures for economic progress; but they are of no use to theoretical economists who are trying to find out how the system works, because they have no significance for conduct.”*

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- Stay tuned for the Friday lecture.

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- It is advocated as a model for “causal analysis” by economists who don’t know much economics.

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- This difference in detail corresponds to the differing objectives of the two approaches.

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- This was the goal of Monograph 10.

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- Haavelmo's distinction is the basis for Pearl's 2000 book on causality that generalizes Haavelmo's analysis to nonlinear settings.

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- This is an “all causes” model in which  $(X, U)$  determine  $Y$ . The variation generated by the hypothetical model varies one coordinate of  $(X, U)$ , fixing all other coordinates to produce the effect of the variation on the outcome  $Y$ .

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- Since we condition on both  $X$  and  $U$ , there is no further source of variation in  $Y$  in an “all causes” model.



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- This relationship does not generate  $U$ -constant  $(Y, X)$  relationships.

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- Rubin and Neyman offer no model of the choice of which outcome is selected.

- In our notation, Neyman (1923) and Rubin assume (PI-1) and (PI-2), but not (PI-3) or (PI-4), since choice is not modeled.



## The “Rubin Model”

### R-1

$\{Y(s, \omega)\}_{s \in S}$ , a set of counterfactuals defined for ex post outcomes. It does not analyze valuations of outcomes nor does it explicitly specify treatment selection rules, except for contrasting randomization with nonrandomization.

### R-2

(PI-1) Invariance of counterfactuals to the assignment mechanism of treatment.

## The “Rubin Model”

### R-3

*No social interactions or general equilibrium effects (PI-2).*

### R-4

*There is no simultaneity in causal effects, i.e., outcomes cannot cause each other reciprocally.*

Two further implicit assumptions in the application of the model are:

- (P-1) is the only problem of interest.
- Mean causal effects are the only objects of interest.
- No analysis of choice behavior.

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Its signature features are:

- 1 Development of an explicit framework for outcomes, measurements and choice of outcomes where the role of unobservables (“missing variables”) in creating selection problems and justifying estimators is explicitly developed.
- 2 The analysis of subjective evaluations of outcomes and the use of choice data to infer them.

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- 3 The analysis of *ex ante* and *ex post* realizations and evaluations of treatments. This analysis enables analysts to model and identify regret and anticipation by agents. Points 2 and 3 introduce human decision making into the treatment effect literature.

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- 4 Development of models for identifying entire distributions of treatment effects (*ex ante* and *ex post*) rather than just the traditional mean parameters focused on by statisticians. These distributions enable analysts to determine the proportion of people who benefit from treatment, something not attempted in the statistical literature on treatment effects.

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- 6 Models for simultaneous causality.
- 7 Definitions of parameters made without appeals to hypothetical experimental manipulations.
- 8 Clarification of the need for invariance of parameters with respect to classes of manipulations to answer classes of questions. This notion is featured in the early Cowles Commission work. See Marschak (1953), Koopmans et al. (1950) and Hurwicz (1962).

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- These distinctions are very clear in Cowles Monograph 10.

## Table 1: Three distinct tasks arising in the analysis of causal models

Task	Description	Requirements
1	Defining the Set of Hypotheticals or Counterfactuals	A Scientific Theory
2	Identifying Parameters (Causal or Otherwise) from Hypothetical Population Data	Mathematical Analysis of Point or Set Identification
3	Identifying Parameters from Data	Estimation and Testing Theory

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- This statement confuses the act of definition of the causal effect (a purely mental act) with empirical difficulties in estimating it (Steps 1 and 2 in Table 1).
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- Issues of definition and identification are confused.

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## The econometric model vs. the Neyman–Rubin ‘causal’ model

- A major limitation of the Neyman–Rubin model is that it is recursive. It cannot model causal effects of outcomes that occur simultaneously.
- By Cowles Monograph 10 this problem had been solved in econometrics.
- It remains an open problem in statistics.

- Write the standard model of simultaneous equations in terms of parameters  $(\Gamma, B)$ , observables  $(Y, X)$  and unobservables  $U$  as

$$\Gamma Y + BX = U, \quad E(U) = 0, \quad (3.2)$$

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- Equation systems like (3.2) are sometimes called “structural equations.”



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- Assume the model is complete ( $\Gamma^{-1}$  exists), gives unique  $Y$ .
- Reduced form is  $Y = \Pi X + R$  where  $\Pi = -\Gamma^{-1}B$  and  $R = \Gamma^{-1}U$ .

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- Without restrictions, *ceteris paribus* manipulations associated with the effect of some components of  $Y$  on other components of  $Y$  are not possible within the model.

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$$Y_2 = \alpha_2 + \gamma_{21} Y_1 + \beta_{21} X_1 + \beta_{22} X_2 + U_2. \quad (3.3b)$$

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- Other restrictions possible.



Structure as invariance to a class of modifications

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(Koopmans, Marschak and Hurwicz)

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- This definition requires a precise definition of a policy, a class of policy modifications and specification of a mechanism through which policy operates.
- Treatment effects can be structural for certain classes of modifications.

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- To reconcile the econometric and treatment effect literatures, go back to a neglected but important paper by Marschak (1953) and taught in his 1949 lectures at Chicago in the Cowles Commission.
- Marschak noted that for many specific questions of policy analysis, it is not necessary to identify fully specified economic models that are invariant to classes of policy modifications.
- Implicit was his use of what we would now call decision theory.

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- Forecasting or evaluating policies may only require partial knowledge of the full simultaneous equations system.
- This principle called **Marschak's maxim** in honor of this insight.

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- The goal of policy analysis under this approach is typically restricted to evaluating policies in place and not in forecasting the effects of new policies or the effects of old policies on new environments.

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- The parameter is not designed to evaluate a whole host of other policies.

- Viewed in this light, the treatment effect literature that compares the outcome associated with  $s \in \mathcal{S}$  with the outcome associated with  $s' \in \mathcal{S}$  seeks to recover a causal effect of  $s$  relative to  $s'$ .

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- It is structural for this intervention.
- Marschak's maxim urges analysts to formulate the problem being addressed clearly and to use the minimal ingredients required to solve it.

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- As analysts ask more difficult questions, it is necessary to specify more features of the models being used to address the questions.
- Marschak's maxim is an application of Occam's Razor to policy evaluation.

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- However, considerable progress has been made in relaxing the parametric structure assumed in the early explicitly economic models.

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- This is task 2 in table 1.

## Table 1: Three distinct tasks arising in the analysis of causal models

Task	Description	Requirements
1	Defining the Set of Hypotheticals or Counterfactuals	A Scientific Theory
2	Identifying Parameters (Causal or Otherwise) from Hypothetical Population Data	Mathematical Analysis of Point or Set Identification
3	Identifying Parameters from Data	Estimation and Testing Theory

## Identification problems: determining models from data

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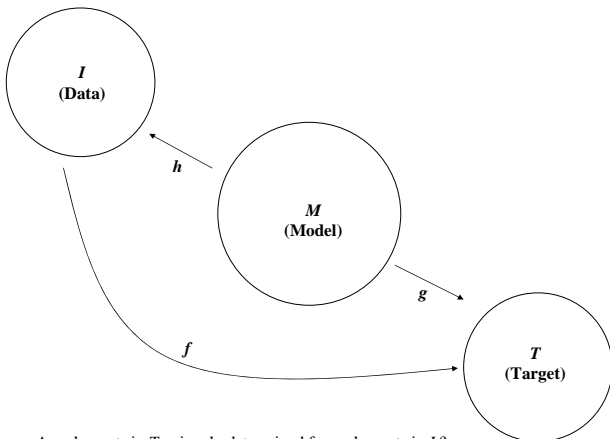
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- Let the class of possible information or data be  $\mathcal{I}$ .
- Define a map  $h : M \rightarrow \mathcal{I}$ .

## Figure 4: Schematic of model ( $M$ ), data ( $\mathcal{I}$ ), and target ( $T$ ) parameter spaces



Are elements in  $T$  uniquely determined from elements in  $I$ ?  
Sometimes  $T = M$ . Usually  $T$  consists of elements derived from  $M$ .

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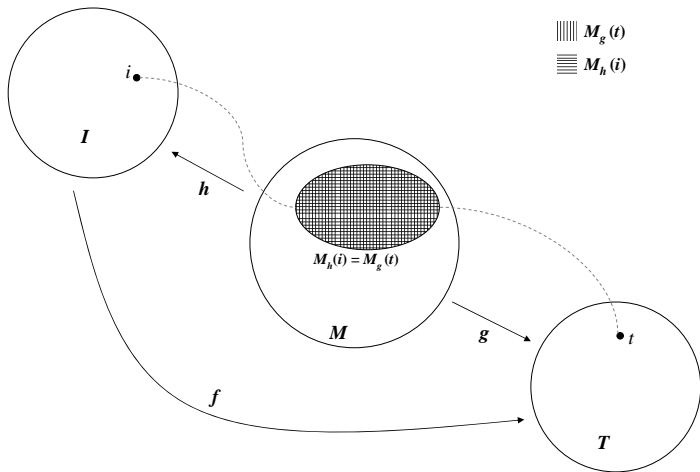
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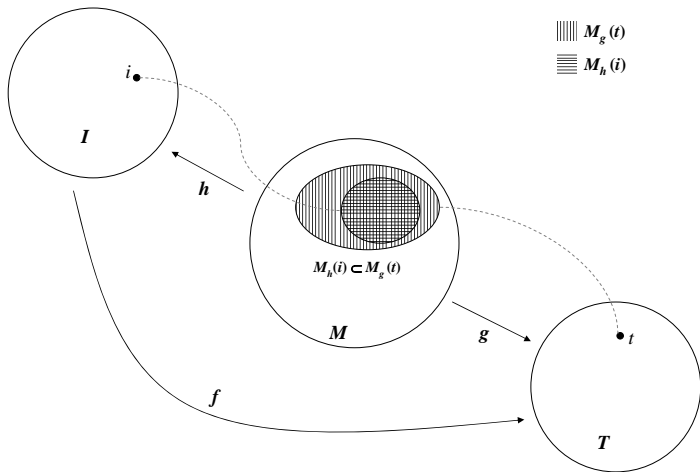
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- $f : \mathcal{I} \rightarrow T$  with the property  $f \circ h = g$  are (a)  $h$  must map  $M$  onto  $\mathcal{I}$  and (b) for all  $i \in \mathcal{I}$ , there exists  $t \in T$  such that  $M_h(i) \subseteq M_g(t)$ .



## Figure 5A: identified model



## Figure 5B: nonidentified model



## Table 3: sources of identification problems

- (i) Absence of data on  $Y(s', \omega)$  for  $s' \in S \setminus \{s\}$  where  $s$  is the state selected (the evaluation problem).
- (ii) Nonrandom selection of observations on states (the selection problem).
- (iii) Support conditions may fail (outcome distributions for  $F(Y_s | X = x)$  may be defined on only a limited support of  $X$  so  $F(X | D_s = 1)$  and  $F(X | D_{s'} = 1)$  have different  $X$  supports or limited overlap in their supports).
- (iv) Functional forms of outcome equations and distributions of unobservables may be unknown. To extend some function  $Y = G(X)$  to a new support requires functional structure: Cannot be extended outside of sample support by a purely nonparametric procedure.
- (v) Determining the  $(X, Z, W)$  conditioning variables.
- (vi) Different information sets for the agent making selection  $\mathcal{I}_a$  and the econometrician trying to identify the model  $\mathcal{I}_e$  where  $\mathcal{I}_a \neq \mathcal{I}_e$ .

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## Two paths toward relaxing distributional, functional form and exogeneity assumptions

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- This is an application of Marschak's maxim.
- But if you focus on one parameter, you should justify why it is interesting.

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- The literature in microeconomic structural estimation focuses on relaxing the linearity, separability, normality and exogeneity conditions invoked in the early literature in order to identify parameters under much weaker conditions.

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- These variations in treatments are taken as the invariant structural parameters.
- The class of modifications considered is the set of treatments in place.

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- The assumptions of the founding fathers have been relaxed in many ways and its methods extended, but the vision is still relevant.
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- My next two lectures demonstrate one way to do this.

