

Randomized Evaluations

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Introduction

- Two cases:
 - 1 A classical argument in experimental design.
 - 2 Other case focuses on solving endogeneity and self-selection problems.
- Randomization is an instrumental variable.

$$Y = \alpha X + \beta D + U \quad (1.1)$$

Introduction

- The case for social experiments that receives the most attention focuses on the dependence between (X, D) and U .

Banerjee (2006):

The beauty of randomized evaluations is that the results are what they are: we compare the outcome in the treatment [group] with the outcome in the control group, see whether they are different, and if so by how much. Interpreting quasi-experiments sometimes requires statistical legerdemain, which makes them less attractive...

Randomization as an instrumental variable

- Treatment choice: Generalized Roy model

$$D = \mathbf{1}(Y_1 - Y_0 - C \geq 0)$$

$$Y_1 = \mu_1(X) + U_1, \quad Y_0 = \mu_0(X) + U_0$$

$$C = \mu_C(W) + U_C, \quad V = U_1 - U_0 - U_C$$

$$\mu_I(X, W) = \mu_1(X) - \mu_0(X) - \mu_C(W), \quad Z = (X, W)$$

Randomization as an instrumental variable

- $\xi = 1$ if an agent is eligible to participate in the program.
- $\xi = 0$ otherwise.
- $\tilde{\xi} = \{0, 1\}$.
- Actual participation A :

$$A = D\xi. \tag{2.1}$$

- Desired participation of the agent (D).

Randomization as an instrumental variable

- Two types of randomization of eligibility.

Randomization of Type 1

A random mechanism (possibly conditional on (X, Z)) is used to determine ξ . The probability of eligibility is $\Pr(\xi = 1 \mid X, Z)$.

Randomization as an instrumental variable

In the context of the generalized Roy model,

(e-1a)

$$\xi \perp\!\!\!\perp (U_0, U_1, U_C) \mid X, Z$$

and

(e-1b)

$\Pr(A = 1 \mid X, Z, \xi)$ depends on ξ .

- Does not impose exogeneity on X, Z .

Randomization as an instrumental variable

In LATE-like notation, define $A(z, e)$ to be the value of A when we set $Z = z$ and $\xi = e$.

Assumption (e-1) is:

(e-1a)'

$$\xi \perp\!\!\!\perp \left(Y_0, Y_1, \{A(z, e)\}_{(z,e) \in \mathcal{Z} \times \tilde{\xi}} \right) \mid X, Z$$

and

(e-1b)'

$\Pr(A = 1 \mid X, Z, \xi)$ depends on ξ .

Randomization as an instrumental variable

- Second type of randomization that randomizes conditional on revealed preference about D .

Randomization of Type 2:

Eligibility may be a function of D as well (conditionally on some or all components of X, Z, Q or unconditionally). It is common to deny entry into programs among people who applied and were accepted into the program ($D = 1$) so the probability of eligibility is $\Pr(\xi = 1 \mid X, Z, Q, D = 1)$. This assumes invariance to randomization.

Randomization as an instrumental variable

For this type of randomization, it is assumed that

(e-2a)

$$\xi \perp\!\!\!\perp (U_0, U_1) \mid X, Z, Q, D = 1 \quad (\text{IV})$$

and

(e-2b)

$$\begin{aligned} \Pr(A = 1 \mid X, Z, D = 1, \xi = 1) &= 1; \\ \Pr(A = 1 \mid X, Z, D = 1, \xi = 0) &= 0. \end{aligned} \quad (\text{rank})$$

- Full compliance.

Randomization as an instrumental variable

Alternatively, we may write:

(e-2a)'

$$\xi \perp\!\!\!\perp (Y_0, Y_1) \mid X, Z, Q, D = 1 \quad (\text{IV})$$

(e-2b)'

$$\begin{aligned} \Pr(A = 1 \mid X, Z, D = 1, \xi = 1) &= 1; \\ \Pr(A = 1 \mid X, Z, D = 1, \xi = 0) &= 0. \end{aligned} \quad (\text{rank})$$

- Full compliance.

What does randomization identify?

- Under (e-1) or equivalently (e-1)' (Randomization Type 1).
- Agents made eligible for the program self-select as usual.
- For those made ineligible we observe y_0 :

$$\begin{aligned} F_0(y_0 | X) &= F_0(y_0 | X, D = 0) \Pr(D = 0 | X) \\ &\quad + F_0(y_0 | X, D = 1) \Pr(D = 1 | X). \end{aligned}$$

- Know $F_0(y_0 | X, D = 0)$ and $\Pr(D = 1 | X)$ from the eligible population.
- Can identify $F_0(y_0 | X, D = 1)$.

What does randomization identify?

- Can identify the TT, $E(Y_1 - Y_0 | X, D = 1)$.
- Cannot identify:
 - ATE ($= E(Y_1 - Y_0 | X)$)
 - $F_{0,1}(y_0, y_1 | X)$
 - $F_{0,1}(y_0, y_1 | X, D = 1)$

What does randomization identify?

- If ξ is a valid instrument for A , we can form the Wald estimand:

$$\begin{aligned} IV_{(e-1)} & \qquad \qquad \qquad (3.1) \\ & = \frac{E(Y \mid \xi = 1, Z, X) - E(Y \mid \xi = 0, Z, X)}{\Pr(A = 1 \mid \xi = 1, Z, X) - \Pr(A = 1 \mid \xi = 0, Z, X)} \end{aligned}$$

- Assuming full compliance,

$$\Pr(A = 1 \mid \xi = 0, Z, X) = 0.$$

What does randomization identify?

Therefore,

$$\begin{aligned} E(Y | \xi = 0, Z, X) & \\ &= E(Y_0 | Z, X) \\ &= E(Y_0 | D = 1, X, Z) \Pr(D = 1 | X, Z) \\ &\quad + E(Y_0 | D = 0, X, Z) \Pr(D = 0 | X, Z). \end{aligned}$$

What does randomization identify?

$$\begin{aligned} E(Y \mid \xi = 1, Z, X) \\ &= E(Y_1 \mid D = 1, Z, X) \Pr(D = 1 \mid Z, X) \\ &\quad + E(Y_0 \mid D = 0, Z, X) \Pr(D = 0 \mid Z, X) \end{aligned}$$

$$IV_{(e-1)} = E(Y_1 - Y_0 \mid D = 1, Z, X).$$

- Randomization does not identify the other mean treatment effects (LATE and ATE) unless the common coefficient model governs the data or $(Y_1 - Y_0)$.

What does randomization identify?

- Since $F(y | X) = E(\mathbf{1}(Y \leq y) | X)$, $IV_{(e-1)}$ also identifies $F_0(y_0 | X, D = 1)$.

What does randomization identify?

- Second type of eligibility randomization proceeds conditionally on $D = 1$.
- Do not identify choice probabilities ($\Pr(D = 1 | X, Z)$).

What does randomization identify?

Under assumption (e-2) or (e-2)':

$$IV_{(e-2)} = \frac{E(Y | D = 1, \xi = 1, X, Z) - E(Y | D = 1, \xi = 0, X, Z)}{\Pr(A = 1 | D = 1, \xi = 1, X, Z) - \Pr(A = 1, D = 1, \xi = 0, X, Z)}.$$

$$\Pr(A = 1 | D = 1, \xi = 1, X, Z) = 1, \text{ and}$$

$$\Pr(A = 1 | D = 1, \xi = 0, X, Z) = 0,$$

$$E(Y | A = 0, D = 1, \xi = 0, X, Z) = E(Y_0 | D = 1, X, Z), \text{ and}$$

$$E(Y | A = 1, D = 1, \xi = 1, X, Z) = E(Y_1 | D = 1, X, Z).$$

- Thus,

$$IV_{(e-2)} = E(Y_1 - Y_0 | D = 1, X, Z).$$

Randomization bias

- Randomization may affect the program being evaluated and change the behavior of participants.
- Can provide “internal validity” on the altered program.
- Has no external validity in this case.

Percentage of local JTPA agencies citing specific concerns about participating in the experiment

Concern	Percentage of Training Centers Citing the Concern
1. Ethical and public relations implications of:	
a. Random assignment in social programs	61.8
b. Denial of services to controls	54.4
2. Potential negative effect of creation of a control group on achievement of client recruitment goals	47.8
3. Potential negative impact on performance standards	25.4
4. Implementation of the study when service providers do intake	21.1
Sample size	228

Percentage of local JTPA agencies citing specific concerns about participating in the experiment

Concern	Percentage of Training Centers Citing the Concern
5. Objections of service providers to the study	17.5
6. Potential staff administrative burden	16.2
7. Possible lack of support by elected officials	15.8
8. Legality of random assignment and possible grievances	14.5
9. Procedures for providing controls with referrals to other services	14.0
10. Special recruitment problems for out-of-school youth	10.5
Sample size	228

Percentage of local JTPA agencies citing specific concerns about participating in the experiment

Notes: Concerns noted by fewer than 5 percent of the training centers are not listed. Percentages may add up to more than 100.0 because training centers could raise more than one concern.

Source: Based on responses of 228 local JTPA agencies contacted about possible participation in the National JTPA Study. From Doolittle and Trager (1990).

Randomization bias

- The parameter $ATE(X) = E(Y_1 - Y_0 | X)$ is the same in the ongoing program as in the population generated by the randomized trial.
- However, treatment parameters conditional on choices such as $TT(X) = E(Y_1 - Y_0 | X, D = 1)$, $TUT(X) = E(Y_1 - Y_0 | X, D = 0)$ are not.
- Analysis applies with full force to LATE.

Randomization bias

- In a model with essential heterogeneity treatment parameters defined conditional on choices are not invariant to the choice of randomization.
- Can still answer P-1, but for the modified program.

Compliance

- The problem of noncompliance.
- Persons assigned to a treatment may not accept it.
- Let $\xi = 1$ if a person is assigned to treatment.
- $\xi = 0$ otherwise.
- Compliance is said to be perfect when $\xi = 1 \Rightarrow A = 1$ and $\xi = 0 \Rightarrow A = 0$.

Compliance

- Noncompliance is a problem if the goal of the social experiment is to estimate $ATE(X) = E(Y_1 - Y_0 | X)$ without using econometric methods to adjust the experimental data.



The dynamics of dropout and program participation

- Stylized multiple stage program.
- In stage “0”, the agent (possibly in conjunction with program officials) decides to participate or not to participate in the program.
- Let $D_0 = 1$ denote that the agent does not choose to participate.
- $D_j = 1, j > 0$, means that the agent is participating through stage j .



The dynamics of dropout and program participation

- Let $\{D_j(z)\}_{z \in \mathcal{Z}}$ be the set of potential treatment choices for choice j associated with setting $Z = z$.
- For each $Z = z$, $\sum_{j=0}^J D_j(z) = 1$.
- Array the collections of choice indicators evaluated at each $Z = z$ into a vector

$$D(z) = (\{D_1(z)\}_{z \in \mathcal{Z}}, \dots, \{D_J(z)\}_{z \in \mathcal{Z}}).$$

$$Y_j = \mu_j(X, U_j), \quad j = 0, \dots, J.$$

- Y_0 is the no treatment state, and the $Y_j, j \geq 1$, correspond to outcomes associated with dropping out at various stages of the program.

The dynamics of dropout and program participation

- In the absence of randomization, the observed Y is

$$Y = \sum_{j=0}^J D_j Y_j.$$

- $\tilde{Y} = (Y_0, \dots, Y_J)$, the vector of potential outcomes.

The dynamics of dropout and program participation

- Let $\xi_j = 1$ denote whether the person is eligible to move beyond stage j .
- $\xi_j = 0$ means the person is randomized out of the program after completing stage j .
- A randomization at stage j with $\xi_j = 1$ means the person is allowed to continue on to stage $j + 1$, although the agent may still choose not to.
- We set $\xi_J \equiv 1$ to simplify the notation.
- $\xi_j = 1$ only if $\xi_\ell = 1, \ell = 0, \dots, j - 1$.
- Array the ξ_j into a vector ξ and denote its support by $\tilde{\xi}$.

The dynamics of dropout and program participation

- A person who does not choose to participate at stage j cannot be forced to do so.
- For a person who would choose k ($D_k = 1$) in a nonexperimental environment, Y_k is observed if $\prod_{\ell=0}^k \xi_{\ell} = 1$.
- Otherwise, if $\xi_k = 0$ but, say, $\prod_{\ell=0}^{k'} \xi_{\ell} = 1$ and $\prod_{\ell=0}^{k'+1} \xi_{\ell} = 0$ for $k' < k$, we observe $Y_{k'}$ for the agent.

The dynamics of dropout and program participation

- From an experiment with randomization administered at different stages, we observe

$$Y = \sum_{j=0}^J D_j \left(\sum_{k=0}^j \left(\prod_{\ell=0}^{k-1} \xi_{\ell} \right) (1 - \xi_k) Y_k \right).$$

- To understand this formula, consider a program with three stages ($J = 3$) after the initial participation stage.
- For a person who would like to complete the program ($D_3 = 1$), but is stopped by randomization after stage 2, we observe Y_2 instead of Y_3 .

The dynamics of dropout and program participation

- Let A_k be the indicator that we observe the agent with a stage k outcome.
- Happens if a person would have chosen to stop at stage k ($D_k = 1$).
- Express A_k as

$$A_k = D_k \prod_{\ell=0}^{k-1} \xi_\ell + \sum_{j \geq k} D_j \left(\prod_{\ell=0}^{k-1} \xi_\ell \right) (1 - \xi_k), \quad k = 0, \dots, J.$$

- If a person who chooses $D_k = 1$ survives all stages of randomization through $k - 1$, we observe Y_k for that person.
- For persons who would choose $D_j = 1, j > k$, but get randomized out at k , we also observe Y_k .

The dynamics of dropout and program participation

- Let $A_i(z, e_i)$ be the value of A_i when $Z = z$ and $\xi_i = e_i$.
- Array the A_i , $i = 1, \dots, J$, into a vector

$$A(z, e) = (A_1(z, e_1), A_2(z, e_2), \dots, A_J(z, e_J)).$$

- $\tilde{Y} = (Y_0, \dots, Y_J)$.

The dynamics of dropout and program participation

- IV conditions for ξ are satisfied under the following sequential randomization assumptions.

(e-3a)

$$\xi_i \perp\!\!\!\perp \left(\tilde{Y}, \{A(z, e)\}_{(z, e) \in \mathcal{Z} \times \tilde{\xi}} \right) \mid X, Z, D_\ell = 1 \text{ for } \ell < i, \\ \prod_{\ell=0}^{i-1} \xi_\ell = 1, \text{ for } i = 0, \dots, J,$$

and

(e-3b)

$$\Pr \left(A_i = 1 \mid X, Z, D_\ell = 1 \text{ for } \ell < i, \prod_{\ell=0}^{i-1} \xi_\ell = 1 \right) \\ \text{depends on } \xi_i, \text{ for } i = 1, \dots, J.$$

The dynamics of dropout and program participation

- To fix ideas, consider a randomization of eligibility ξ_0 , setting $\xi_1 = \dots = \xi_J = 1$.
- For those declared eligible,

$$E(Y | \xi_0 = 1) = \sum_{j=0}^J E(Y_j | D_j = 1) \Pr(D_j = 1). \quad (6.1)$$

- For those declared ineligible,

$$E(Y | \xi_0 = 0) = \sum_{j=0}^J E(Y_0 | D_j = 1) \Pr(D_j = 1). \quad (6.2)$$

The dynamics of dropout and program participation

- From observed choice behavior we can identify each of the components of (6.1).
- We observe $\Pr(D_j = 1)$ from observed choices of treatment, and we observe $E(Y_j | D_j = 1)$ from observed outcomes for each treatment choice.
- The individual components of (6.2) apart from the probabilities cannot, without further assumptions, be identified by the experiment.

The dynamics of dropout and program participation

$$\begin{aligned} E(Y \mid \xi_0 = 1) - E(Y \mid \xi_0 = 0) & \quad (6.3) \\ &= \sum_{j=1}^J E(Y_j - Y_0 \mid D_j = 1) \Pr(D_j = 1) \end{aligned}$$

- For $J > 1$, this simple experimental estimator does not identify the effect of full participation in the program for those who participate ($E(Y_j - Y_0 \mid D_j = 1)$) unless partial participation has the same mean effect as full participation for persons who drop out at the early stages.

The dynamics of dropout and program participation

- More generally, suppose we randomize persons out after completing stage k ($[\prod_{\ell=0}^{k-1} \xi_{\ell}] (1 - \xi_k) = 1$).
- For another group establish full eligibility at all stages ($\prod_{\ell=0}^J \xi_{\ell} = 1$), we obtain

$$\begin{aligned}
 & E \left[Y \mid \prod_{\ell=0}^J \xi_{\ell} = 1 \right] - E \left[Y \mid \left(\prod_{\ell=0}^{k-1} \xi_{\ell} \right) (1 - \xi_k) = 1 \right] \\
 &= \sum_{j=k}^J E(Y_j - Y_k \mid D_j = 1) \Pr(D_j = 1).
 \end{aligned}$$

The dynamics of dropout and program participation

- Hence, since we know $E(Y_k | D_k = 1)$ and $\Pr(D_k = 1)$ we can identify only

$$\sum_{j=k+1}^J E(Y_k | D_j = 1) \Pr(D_j = 1). \quad (6.4)$$

The dynamics of dropout and program participation

- Observe that a randomization of eligibility that prevents people from going to stage $J - 1$ but not to stage J ($(\prod_{\ell=0}^{J-2} \xi_{\ell}) (1 - \xi_{J-1}) = 1$) identifies $E(Y_J - Y_{J-1} | D_J = 1)$:

$$\begin{aligned}
 & E(Y | \xi_0 = 1, \dots, \xi_{J-2} = 1, \xi_{J-1} = 0) \\
 &= \left[\sum_{j=0}^{J-1} E(Y_j | D_j = 1) \Pr(D_j = 1) \right] \\
 &\quad + E(Y_{J-1} | D_J = 1) \Pr(D_J = 1).
 \end{aligned}$$

The dynamics of dropout and program participation

- Thus we identify

$$\left\{ \begin{array}{l} E(Y | \xi_0 = 1, \dots, \xi_J = 1) \\ -E(Y | \xi_0 = 1, \dots, \xi_{J-1} = 1, \xi_J = 0) \end{array} \right\} \\ = E(Y_J - Y_{J-1} | D_J = 1) \Pr(D_J = 1),$$

and hence $E(Y_J - Y_{J-1} | D_J = 1)$.

The dynamics of dropout and program participation

$$E(Y \mid \xi_0 = 1, \dots, \xi_{\ell-1} = 1, \xi_{\ell} = 0)$$

all components known from observational data

$$= \sum_{j=0}^{\ell} E(Y_j \mid D_j = 1) \Pr(D_j = 1) + \underbrace{\sum_{j=\ell+1}^J E(Y_{\ell} \mid D_j = 1) \Pr(D_j = 1)}_{\text{sum and probability weights known, but not individual } E(Y_{\ell} \mid D_j = 1)}$$

sum and probability weights known,

but not individual $E(Y_{\ell} \mid D_j = 1)$



Parameters and combinations of parameters that can be identified by different randomizations

Choice Probabilities
(known)

Outcome

		Y_0	Y_1	...	Y_j	...	Y_{j-1}	Y_j
$\Pr(D_0 = 1)$	D_0	$E(Y_0 D_0 = 1)$	$E(Y_1 D_0 = 1)$...	$E(Y_j D_0 = 1)$...	$E(Y_{j-1} D_0 = 1)$	$E(Y_j D_0 = 1)$
$\Pr(D_1 = 1)$	D_1	$E(Y_0 D_1 = 1)$	$E(Y_1 D_1 = 1)$...	$E(Y_j D_1 = 1)$...	$E(Y_{j-1} D_1 = 1)$	$E(Y_j D_1 = 1)$
$\Pr(D_2 = 1)$	D_2	$E(Y_0 D_2 = 1)$	$E(Y_1 D_2 = 1)$...	$E(Y_j D_2 = 1)$...	$E(Y_{j-1} D_2 = 1)$	$E(Y_j D_2 = 1)$
$\Pr(D_j = 1)$	D_j	\vdots	\vdots		\vdots		\vdots	\vdots
		$E(Y_0 D_j = 1)$	$E(Y_1 D_j = 1)$...	$E(Y_j D_j = 1)$...	$E(Y_{j-1} D_j = 1)$	$E(Y_j D_j = 1)$
		\vdots	\vdots		\vdots		\vdots	\vdots
$\Pr(D_{j-1} = 1)$	D_{j-1}	$E(Y_0 D_{j-1} = 1)$	$E(Y_1 D_{j-1} = 1)$...	$E(Y_j D_{j-1} = 1)$...	$E(Y_{j-1} D_{j-1} = 1)$	$E(Y_j D_{j-1} = 1)$
$\Pr(D_j = 1)$	D_j	$E(Y_0 D_j = 1)$	$E(Y_1 D_j = 1)$...	$E(Y_j D_j = 1)$...	$E(Y_{j-1} D_j = 1)$	$E(Y_j D_j = 1)$

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Randomization

$\xi_0 = 0$

$\xi_1 = 0$

...

$\xi_j = 0$

...

$\xi_{j-1} = 0$

$\xi_j = 0$

New Identified
Combinations of
Parameters

$$\sum_{\ell=1}^j \{E(Y_0 | D_\ell = 1) \times \Pr(D_\ell = 1)\}$$

$$\sum_{\ell=2}^j \{E(Y_1 | D_\ell = 1) \times \Pr(D_\ell = 1)\}$$

...

$$\sum_{\ell=j+1}^j \{E(Y_j | D_\ell = 1) \times \Pr(D_\ell = 1)\}$$

...

$$E(Y_{j-1} | D_j = 1)$$

The dynamics of dropout and program participation

- Randomization at stage ℓ is an IV.

$$Y = \sum_{j=0}^J A_j Y_j.$$



The dynamics of dropout and program participation

$$\begin{aligned}
 IV_{\xi_\ell} &= \frac{E[Y | \xi_\ell = 0] - E[Y | \xi_\ell = 1]}{\Pr(A_\ell = 1 | \xi_\ell = 0) - \Pr(A_\ell = 1 | \xi_\ell = 1)} \\
 &= \frac{\sum_{j=\ell+1}^J E[Y_\ell - Y_j | D_j = 1] \Pr(D_j = 1)}{\sum_{j=\ell+1}^J \Pr(D_j = 1)}, \quad \ell = 0, \dots, J - 1.
 \end{aligned}$$

Dropout and Substitution Bias

Study	Authors/time period	Target group(s)	Fraction of treatments receiving services	Fraction of controls receiving services
1. NSW	Hollister, et al. (1984) (9 months after RA)	Long-term AFDC women	0.95	0.11
		Ex-addicts	NA	0.03
		17-20 year old high school dropouts	NA	0.04
2. SWIM	Friedlander and Hamilton (1993) (Time period not reported)	AFDC women: applicants and recipients		
		a. Job search assistance	0.54	0.01
		b. Work experience	0.21	0.01
		c. Classroom training/OJT	0.39	0.21
		d. Any activity	0.69	0.30
		AFDC-U unemployed fathers		
		a. Job search assistance	0.60	0.01
		b. Work experience	0.21	0.01
c. Classroom training/OJT	0.34	0.22		
d. Any activity	0.70	0.23		

Note: RA = random assignment

Dropout and Substitution Bias

Study	Authors/time period	Target group(s)	Fraction of treatments receiving services	Fraction of controls receiving services
3. JOBSTART	Cave, et al. (1993) (12 months after RA)	Youth high school dropouts Classroom training/OJT	0.90	0.26
4. Project	Kemple, et al. (1995) (24 months after RA)	AFDC women: applicants and recipients a. Job search assistance b. Classroom training/OJT c. Any activity	0.43 0.42 0.64	0.19 0.31 0.40

Note: RA = random assignment

Dropout and Substitution Bias

Study	Authors/time period	Target group(s)	Fraction of treatments receiving services	Fraction of controls receiving services
5. New Chance	Quint, et al. (1994) (18 months after RA)	Teenage single mothers		
		Any education services	0.82	0.48
		Any training services	0.26	0.15
		Any education or training	0.87	0.55
6. National JTPA Study	Heckman and Smith (1998) (18 months after RA)	Self-reported from survey data		
		Adult males	0.38	0.24
		Adult females	0.51	0.33
		Male youth	0.50	0.32
		Female youth	0.81	0.42
		Combined Administrative Survey Data		
		Adult males	0.74	0.25
		Adult females	0.78	0.34
		Male youth	0.81	0.34
		Female youth	0.81	0.42

Note: RA = random assignment