The Technology of Skill Formation

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This paper uses simple economic models of skill formation to organize a large body of evidence on the development of skills in children in economics, psychology, education and neuroscience.

The existing theoretical literature on child development in economics treats childhood as a single period (see, e.g., Gary S. Becker and Nigel Tomes, 1986; Roland Benabou, 2000; S. Rao Aiyagari et al., 2002).

The implicit assumption in this approach is that inputs into the production of skills at different stages of childhood are perfect substitutes.
To account for a large body of evidence, it is important to build models of skill formation with multiple stages of childhood, where inputs at different stages are complements and where there is self-productivity of investment.

In order to rationalize the evidence, it is also important to recognize three distinct credit constraints operating on the family and its children.

1. The inability of a child to choose its parents. This is the fundamental constraint imposed by the accident of birth.
2. The inability of parents to borrow against their children’s future income to finance investments in them.
3. The inability of parents to borrow against their own income to finance investments in their children.
A model that is faithful to the evidence must recognize:

(a) Parental influences are key factors governing child development;

(b) Early childhood investments must be distinguished from late childhood investments;

(c) An equity-efficiency trade-off exists for late investments, but not for early investments;

(d) Abilities are created, not solely inherited, and are multiple in variety;

(e) The traditional ability-skills dichotomy is misleading. Both skills and abilities are created; and

(f) The “nature versus nurture” distinction is obsolete.
• These insights change the way we interpret evidence and design policy about investing in children.
• Point (a) is emphasized in many papers.
• Point (b) is ignored in models that consider only one period of childhood investment.
• Points (c), (d) and (e) have received scant attention in the formal literature on child investment.
• Point (f) is ignored in the literature that partitions the variance of child outcomes into components due to nature and components due to nurture.
Observations About Human Diversity and Human Development and Some Facts Our Model Explains
Three Observations

- The first observation is that *ability matters*.
- A large number of empirical studies document that cognitive ability is a powerful determinant of wages, schooling, participation in crime and success in many aspects of social and economic life.
Second observation: *Abilities are multiple in nature.*

Noncognitive abilities (perseverance, motivation, time preference, risk aversion, self-esteem, self-control, preferences for leisure) have direct effects on wages (controlling for schooling), schooling, teenage pregnancy, smoking, crime, performance on achievement tests and many other aspects of social and economic life.

See, e.g., Samuel Bowles and Herb Gintis (1976); Samuel Bowles et al. (2001); Lex Borghans et al. (2006); Heckman et al. (2006).
A third observation is that the “nature versus nurture” distinction is obsolete.

The modern literature on epigenetic expression teaches us that the sharp distinction between acquired skills and ability featured in the early human capital literature is not tenable.

Additive “nature” and “nurture” models, while traditional and still used in many studies of heritability and family influence, mischaracterize how ability is manifested.
Abilities are produced, and gene expression is governed by environmental conditions (Eric Turkheimer et al., 2003).

Measured abilities are susceptible to environmental influences, including in utero experiences, and also have genetic components.

These factors interact to produce abilities that have both a genetic and an acquired character and the modified genes are heritable. Genes and environment cannot be meaningfully parsed by traditional linear models that assign variance to each component, even though it is traditional to do so.
Six Facts We Explain

- First, *ability gaps between individuals and across socioeconomic groups open up at early ages, for both cognitive and noncognitive skills.*
- Adjusting for family background by regression analysis reduces these gaps.
- Experimental manipulations of early environments (Perry, Abecedarian et al.) show that these effects are causal.
Figure 1: Children of NLSY Average Standardized Score PIAT Math by Permanent Income Quartile

Source: Full Sample of CNLSY
Six Facts We Explain

Figure D0: Trend in Mean Cognitive Score by Maternal Education

The dramatic results on the importance of the early years in creating differences among children shown in the previous graph arise if “Bayley scores” are used as a measure of cognition at age 1.

As Michael Lewis and Harry McGurk (1972) point out, this is illegitimate since the Bayley score tests other aspects of child development in addition to cognition.
Six Facts We Explain

Figure D00: Children of NLSY Average Standardized Score
Peabody Picture Vocabulary Test by Permanent Income Quartile

Source: Full Sample of CNLSY
Six Facts We Explain

Figure D1a. Average percentile rank on PIAT-Math score, by income quartile
Six Facts We Explain

Figure D1b. Adjusted average PIAT-Math score percentiles, by income quartile

*Residualized on maternal education, maternal AFQT (corrected for the effect of schooling) and broken home at each age
Six Facts We Explain

Figure D2a. Average percentile rank on PIAT-Math score, by race
Figure D2b. Adjusted average PIAT-Math score percentiles, by race

* Residualized on maternal education, maternal AFQT (corrected for the effect of schooling) and broken home at each age

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**Six Facts We Explain**

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**Observations**

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**Model**

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**Optimal Investment**

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**Cog and Noncog Estimates**

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**Lessons**

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**Summary**

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Six Facts We Explain

Figure D3a. Average percentile rank on anti-social behavior score, by income quartile
Six Facts We Explain

Figure D3b. Adjusted average anti-social behavior score percentile, by income quartile

* Residualized on maternal education, maternal AFQT (corrected for the effect of schooling) and broken home at each age
Figure D4a. Average percentile rank on anti-social behavior score, by race
Six Facts We Explain

Figure D4b. Adjusted average anti-social behavior score percentile, by race

* Residualized on maternal education, maternal AFQT (corrected for the effect of schooling) and broken home at each age
Six Facts We Explain

Figure D5a. Early Childhood Longitudinal Study (ECLS) Reading

Source: Raudenbush (2006)
Six Facts We Explain

Figure D5b. Mean trajectories, high and low priority schools (ECLS) Math

Source: Raudenbush (2006)
Six Facts We Explain

Figure D6a. Average Trajectories, Grades 1–3, high and low poverty schools (Sustaining Effects Study)

Reading

Source: Raudenbush (2006)
Six Facts We Explain

Figure D6b. Average Trajectories, Grades 1–3, high and low poverty schools (Sustaining Effects Study)

Math

Source: Raudenbush (2006)
Figure D7a. Average achievement trajectories, grades 8–12, (NELS 88)

Science

Source: Raudenbush (2006)
Six Facts We Explain

Figure D7b. Average achievement trajectories, grades 8–12, (NELS 88) Math

Source: Raudenbush (2006)
Figure D8a. Growth as a function of student social background: ECLS Reading

Source: Raudenbush (2006)
Figure D8b. Growth as a function of student social background: ECLS Math

Source: Raudenbush (2006)
Six Facts We Explain

**Figure D9a. Growth as a function of school poverty for poor children: sustaining effects data**

Reading

Source: Raudenbush (2006)
Six Facts We Explain

Figure D9b. Growth as a function of school poverty for poor children: sustaining effects data
Math

Source: Raudenbush (2006)
Schooling quality and school resources have relatively small effects on ability deficits and only marginally account for any divergence by age across children from different socioeconomic groups in test scores.

Six Facts We Explain

- Second, *in both animal and human species, there is compelling evidence of critical and sensitive periods in the development of the child.*
Second language learning

![Graph showing English proficiency scores for different ages of arrival in the U.S.](image)
Six Facts We Explain

- The later remediation is given to a disadvantaged child, the less effective it is.

- A substantial body of evidence suggests that returns to adolescent education for the most disadvantaged and less able are lower than the returns for the more advantaged.
Six Facts We Explain

- The economic returns to adolescent interventions—job training, high school graduation, and college attendance—are lower for less able persons.
Six Facts We Explain

Table 1. Return to one year of college for individuals at different percentiles of the math test score distribution
White males from high school and beyond

<table>
<thead>
<tr>
<th></th>
<th>5%</th>
<th>25%</th>
<th>50%</th>
<th>75%</th>
<th>95%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average return in the population</td>
<td>0.1121</td>
<td>0.1374</td>
<td>0.1606</td>
<td>0.1831</td>
<td>0.2101</td>
</tr>
<tr>
<td></td>
<td>(0.0400)</td>
<td>(0.0328)</td>
<td>(0.0357)</td>
<td>(0.0458)</td>
<td>(0.0622)</td>
</tr>
<tr>
<td>Return for those who attend college</td>
<td>0.1640</td>
<td>0.1893</td>
<td>0.2125</td>
<td>0.2350</td>
<td>0.2621</td>
</tr>
<tr>
<td></td>
<td>(0.0503)</td>
<td>(0.0582)</td>
<td>(0.0676)</td>
<td>(0.0801)</td>
<td>(0.0962)</td>
</tr>
<tr>
<td>Return for those who do not attend college</td>
<td>0.0702</td>
<td>0.0954</td>
<td>0.1187</td>
<td>0.1411</td>
<td>0.1682</td>
</tr>
<tr>
<td></td>
<td>(0.0536)</td>
<td>(0.0385)</td>
<td>(0.0298)</td>
<td>(0.0305)</td>
<td>(0.0425)</td>
</tr>
<tr>
<td>Return for those at the margin</td>
<td>0.1203</td>
<td>0.1456</td>
<td>0.1689</td>
<td>0.1913</td>
<td>0.2184</td>
</tr>
<tr>
<td></td>
<td>(0.0364)</td>
<td>(0.0300)</td>
<td>(0.0345)</td>
<td>(0.0453)</td>
<td>(0.0631)</td>
</tr>
</tbody>
</table>

Source: Carneiro and Heckman (2003)
Third, despite the low returns to interventions targeted toward disadvantaged adolescents, the empirical literature shows high economic returns for remedial investments in young disadvantaged children.
Six Facts We Explain

- Fourth, *if early investment in disadvantaged children is not followed up by later investment, its effect tends to weaken at later ages.*

- Currie and Thomas (1995) document a decline in the performance of minority Head Start participants after they leave the program.
Six Facts We Explain

- Fifth, *the effects of credit constraints on the child’s adult outcomes depend on the age at which they bind for the child’s family.*

- Controlling for cognitive ability, under meritocratic policies currently in place in American society, family income during the child’s college-going years plays only a minor role in determining child college participation.

- Holding ability fixed, minorities are *more likely* to attend college than others despite their lower family incomes.
Carneiro and Heckman present evidence for the United States that only a small fraction (at most 8%) of the families of adolescents are credit constrained in making their college decisions.

This evidence is supported in research by Cameron and Taber (2004) and Stinebrickner and Stinebrickner (2006).
There is some evidence that credit constraints operating in the *early* years have effects on adult ability and schooling outcomes, but there is not full agreement in the literature on the magnitude of the effect (Duncan and Brooks-Gunn, 1997; Dahl and Lochner, 2004; Duncan and Ariel Kalil, 2006; Carneiro and Heckman, 2003).
The empirically important market failures in the life cycle of skill formation in contemporary American society are the inability of children to buy their parents or the lifetime resources that parents provide.

It is not credit constraints facing families seeking to secure loans for a child’s education when the child is an adolescent.
Six Facts We Explain

- Sixth, *socioemotional (noncognitive) skills foster cognitive skills and are an important product of successful families and successful interventions in disadvantaged families.*

- The Perry Preschool Program, which was evaluated by random assignment, did not boost participant adult IQ but enhanced performance of participants in a number of dimensions, including elevated scores on achievement tests, employment and reduced participation in a variety of social pathologies.
Six Facts We Explain

Figure D10a. Perry Preschool Program: IQ, by age and treatment group
Six Facts We Explain

Figure D10b. Perry Preschool Program: educational effects, by treatment group

Notes: *High achievement defined as performance at or above the lowest 10th percentile on the California Achievement Test (1970).
Six Facts We Explain

**Figure D10c. Perry Preschool Program: economic effects at age 27, by treatment group**

- **Earn +$2,000 Monthly**
  - Treatment: 29%
  - Control: 7%

- **Own Home**
  - Treatment: 36%
  - Control: 13%

- **Never on Welfare as Adult**
  - Treatment: 29%
  - Control: 14%

Source: Barnett (2004). *Updated through Age 40 using recent Perry Preschool Program data, derived from self-report and all available state records.*
Figure D10d. Perry Preschool Program: arrests per person before age 40, by treatment group

Source: Perry Preschool Program. Juvenile arrests are defined as arrests prior to age 19.
Six Facts We Explain

Table D1. Economic benefits and costs

<table>
<thead>
<tr>
<th></th>
<th>Perry</th>
<th>Chicago CPC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Child Care</td>
<td>986</td>
<td>1,916</td>
</tr>
<tr>
<td>Earnings</td>
<td>40,537</td>
<td>32,099</td>
</tr>
<tr>
<td>K-12</td>
<td>9,184</td>
<td>5,634</td>
</tr>
<tr>
<td>College/Adult</td>
<td>-782</td>
<td>-644</td>
</tr>
<tr>
<td>Crime</td>
<td>94,065</td>
<td>15,329</td>
</tr>
<tr>
<td>Welfare</td>
<td>355</td>
<td>546</td>
</tr>
<tr>
<td>FG Earnings</td>
<td>6,181</td>
<td>4,894</td>
</tr>
<tr>
<td>Abuse/Neglect</td>
<td>0</td>
<td>344</td>
</tr>
<tr>
<td><strong>Total Benefits</strong></td>
<td>150,525</td>
<td>60,117</td>
</tr>
<tr>
<td><strong>Total Costs</strong></td>
<td>16,514</td>
<td>7,738</td>
</tr>
<tr>
<td><strong>Net Present Value</strong></td>
<td>134,011</td>
<td>52,380</td>
</tr>
<tr>
<td><strong>Benefits-To-Costs Ratio</strong></td>
<td>9.11</td>
<td>7.77</td>
</tr>
</tbody>
</table>

Notes: All values discounted at 3% and are in $2004. Numbers differ slightly from earlier estimates because FG Earnings for Perry and Chicago were estimated using the ratio of FG Earnings Effect to Earnings Effect (about 15%) that was found in Abecedarian.

A Model of Skill Formation

- Agents possess a vector of abilities at each age.
- These abilities (or skills) are multiple in nature and range from pure cognitive abilities (e.g. IQ) to noncognitive abilities (patience, self control, temperament, risk aversion, time preference).
- These abilities are used with different weights in different tasks in the labor market and in social life more generally.
The human skill formation process is governed by a multistage technology.

Each stage corresponds to a period in the life cycle of a child.

Like earlier work by Ben-Porath (1967), we use a production function to determine the relationship between inputs and the output of skill.

Unlike Ben-Porath, in our models qualitatively different inputs can be used at different stages and the technologies can be different at different stages of child development.
Ben-Porath focuses on adult investments where time and its opportunity cost play important roles.

For child investments, parents make decisions and child opportunity costs are less relevant.
The outputs at each stage in our technology are the levels of each skill achieved at that stage.

Some stages of the lifecycle may be more productive in producing some skills than other stages, and some inputs may be more productive at some stages than at other stages.

Those stages that are more effective in producing certain skills are called “sensitive periods” for the acquisition of those skills.

If one stage alone is effective in producing a skill (or ability), it is called a “critical period” for that skill.
The skills produced at one stage augment the skills attained at later stages. This effect is termed *self-productivity*.

Skills are self-reinforcing and cross-fertilizing.

A second key feature of skill formation is *dynamic complementarity*.

Skills produced at one stage raise the productivity of investment at subsequent stages. Complementarity implies that levels of skill investments at different ages bolster each other. They are synergistic.
Together, dynamic complementarity and self-productivity produce *multiplier effects* which are the mechanisms through which skills beget skills and abilities beget abilities.

Dynamic complementarity, self-productivity of human capital, and multiplier effects imply an equity-efficiency trade-off for late child investments but not for early investments.

These concepts, embedded in alternative market settings, explain the six facts previously listed.
Overlapping generations model

A Simple Model of Skill Formation

- An individual lives for $2T$ years.
- The first $T$ years the individual is a child of an adult parent.
- From age $T + 1$ to $2T$ the individual lives as an adult and is the parent of a child.
- The individual dies at the end of the period in which he is $2T$ years-old, just before his child’s child is born.
A household consists of an adult parent and his child.

The parents invest in their children because of altruism. They have common preferences and supply labor inelastically.

$l_t$ denotes parental investments in child skill when the child is $t$ years-old, where $t = 1, 2, \ldots, T$.

The output of the investment process is a skill vector.

We ignore investments in the adult years to focus on new ideas in this paper. Government inputs (e.g., schooling) are a component of $l_t$. 
Each agent is born with initial conditions $\theta_1$.

$h$ is the parental characteristics (e.g., their IQ, education, etc.).

$\theta_t$ is the vector of skill stocks.

The technology of production of skill when the child is $t$ years-old is

$$\theta_{t+1} = f_t (h, \theta_t, I_t),$$  \hspace{1cm} (1)

for $t = 1, 2, \ldots, T$.

We assume that $f_t$ is neoclassical: strictly increasing, strictly concave, and twice continuously differentiable.
Substituting (1) for $\theta_t, \theta_{t-1}, \ldots$, repeatedly, one can rewrite the stock of skills at stage $t+1$, $\theta_{t+1}$, as a function of all past investments:

$$\theta_{t+1} = m_t (h, \theta_1, l_1, \ldots, l_t).$$

(2)
Dynamic complementarity arises when

\[
\frac{\partial^2 f_t(h, \theta_t, l_t)}{\partial \theta_t \partial l_t'} > 0.
\]
Self-productivity arises when

$$\frac{\partial f_t (h, \theta_t, l_t)}{\partial \theta_t} > 0.$$ 

For the case of skill vectors, this includes own and cross effects.
This technology describes learning in rodents and macaques as documented, respectively, by Meaney (2001) and Cameron (2004).

Early parental emotional environments encourage the animals to explore (and learn) more.

This technology also captures the critical and sensitive periods in humans and animals.
Period $t^*$ is a critical period for $\theta_{t+1}$ if
\[ \frac{\partial \theta_{t+1}}{\partial l_s} = \frac{\partial m_t (h, \theta_1, l_1, \ldots, l_t)}{\partial l_s} \equiv 0 \text{ for all } \theta_1, l_1, \ldots, l_t, s \neq t^*, \]
but
\[ \frac{\partial \theta_{t+1}}{\partial l_{t^*}} = \frac{\partial m_t (h, \theta_1, l_1, \ldots, l_t)}{\partial l_{t^*}} > 0 \text{ for some } \theta_1, l_1, \ldots, l_t. \]

Investments in $\theta_{t+1}$ are productive in period $t^*$ but not in any other period $s \neq t^*$.

Period $t^*$ is a sensitive period for $\theta_{t+1}$ if
\[ \left. \frac{\partial \theta_{t+1}}{\partial l_s} \right|_{h=h, \theta_1=\theta, l_1=i_1, \ldots, l_t=i_t} < \left. \frac{\partial \theta_{t+1}}{\partial l_{t^*}} \right|_{h=h, \theta_1=\theta, l_1=i_1, \ldots, l_t=i_t}. \]

At the same level of inputs, investment is more productive in stage $t^*$ than in other stage $s \neq t^*$. 
Suppose that $T = 2$.

Assume $\theta_1$, $l_1$, $l_2$ are scalars.

The child’s adult stock of skills, $h’ (= \theta_3)$, is

$$h’ = m_2 (h, \theta_1, l_1, l_2).$$

(3)
The literature in economics assumes only one period of childhood.

It does not distinguish between early investment and later investment.
The conventional specification is a special case of technology (3), where

$$h' = m_2(h, \theta_1, \gamma l_1 + (1 - \gamma) l_2)$$  \hspace{1cm} (4)

and $\gamma = 1/2$.

Adult stocks of skills do not depend on how investments are distributed over different periods of childhood.
• The polar opposite of perfect substitution is perfect complementarity:

\[ h' = m_2 (h, \theta_1, \min \{ I_1, I_2 \}) . \]  

(5)

• Adult stocks of skills critically depend on how investments are distributed over time.

• If investments in period one are zero, \( I_1 = 0 \), then it does not pay to invest in period two.

• If late investments are zero, \( I_2 = 0 \), it does not pay to invest early.
Complementarity has a dual face.

It is essential to invest early to get satisfactory adult outcomes.

But it is also essential to invest late to harvest the fruits of the early investment.
Complementarity helps to explain the summary of the evidence by Currie and Thomas (1995) that for certain groups early investments through Head Start have weak effects in later years if not followed up by later investments.

This explanation is in sharp contrast to the one offered by Becker (1991) that explains the Headstart fadeout by crowding out of parental investments by public investment.

That is a story of substitution in a one-period model of childhood.
More general technology:

\[ h' = m_2 \left( h, \theta_1, \left[ \gamma (I_1) + (1 - \gamma) (I_2) \right]^{\frac{1}{\phi}} \right), \]

for \( \phi \leq 1 \) and \( 0 \leq \gamma \leq 1 \).

The CES share parameter \( \gamma \) is a skill multiplier.

It arises from the productivity of early investment not only in directly boosting \( h' \) (through self-productivity) but also in raising the productivity of \( I_2 \) by increasing \( \theta_2 \) through first period investments.

Thus \( I_1 \) directly increases \( \theta_2 \) which in turn affects the productivity of \( I_2 \) in forming \( h' \).

\( \gamma \) captures the net effect of \( I_1 \) on \( h' \) through both self-productivity and direct complementarity.
Elasticity of substitution $1/(1 - \phi)$ is a measure of how easy it is to substitute between $I_1$ and $I_2$.

$\phi$ represents the degree of complementarity (or substitutability) between early and late investments in producing skills.

When $\phi$ is small, low levels of early investment $I_1$ are not easily remediated by later investment $I_2$ in producing human capital.

The other face of CES complementarity is that when $\phi$ is small, high early investments should be followed with high late investments if the early investments are to be harvested.

In the extreme case when $\phi \to -\infty$, (6) converges to (5).
This technology explains facts two and three — why returns to education are low in the adolescent years for disadvantaged (low $h$, low $l_1$, low $\theta_2$) adolescents but are high in the early years.

In a one-period model of childhood, inputs at any stage of childhood are perfect substitutes.

Application of the one period model supports the widely held but empirically unsupported intuition that diminishing returns make investment in less advantaged adolescents more productive.

The evidence — fact two — suggests that just the opposite is true.
The Optimal Lifecycle Profile of Investments

- We show how the ratio of early to late investments varies as a function of $\phi$ and $\gamma$ as a consequence of parental choice in different market settings.
- Let $w$ and $r$ denote the wage and interest rates, respectively, and assume a stationary environment.
- At the beginning of adulthood, the parents draw the initial level of skill of the child, $\theta_1$, from $J(\theta_1)$.  

Upon reaching adulthood, the parents receive bequest $b$.

State variables for the parent are the parental skills, $h$, the parental financial resources, $b$, and the initial skill level of the child, $\theta_1$.

Let $c_1$ and $c_2$ denote the consumption of the household in the first and second period of the lifecycle of the child.

The budget constraint is:

$$c_1 + l_1 + \frac{c_2 + l_2}{(1 + r)} + \frac{b'}{(1 + r)^2} = wh + \frac{wh}{(1 + r)} + b. \quad (7)$$
\( \beta \) is the utility discount factor
\( \delta \) is a measure of parental altruism toward the child.
\( u(\cdot) \) is the utility function.
The recursive formulation of the problem of the parent is:

\[
V(h, b, \theta_1) = \max \left\{ u(c_1) + \beta u(c_2) + \beta^2 \delta E[V(h', b', \theta_1')] \right\}.
\]  
(8)
When $\phi = 1$, so early and late investments are perfect CES substitutes, the optimal investment strategy is straightforward.

- The price of early investment is $1$.
- The price of the late investment is $1/(1 + r)$.
- Thus the parents can purchase $(1 + r)$ units of $I_2$ for every unit of $I_1$. 
The amount of human capital produced from one unit of \( l_1 \) is \( \gamma \), while \( (1 + r) \) of \( l_2 \) produces \( (1 + r)(1 - \gamma) \) units of human capital.

Thus, two forces act in opposite directions. High productivity of initial investment (the skill multiplier \( \gamma \)) drives the agent toward making early investments.

The interest rate drives the agent to invest late. It is optimal to invest early if \( \gamma > (1 - \gamma)(1 + r) \).
As $\phi \to -\infty$ (perfect complementarity), the optimal investment strategy is to set $I_1 = I_2$.

In this case, investments in the young are essential.

At the same time, later investments are needed to harvest early investments.

On efficiency grounds, early disadvantages should be perpetuated, and compensatory investments at later ages are economically inefficient.
For $-\infty < \phi < 1$, the first-order conditions are necessary and sufficient given concavity of the technology in terms of $I_1$ and $I_2$.

For an interior solution, we can derive the optimal ratio of early to late investments:

$$\frac{l_1}{l_2} = \left[\frac{\gamma}{(1-\gamma)(1+r)}\right]^{\frac{1}{1-\phi}}. \quad (9)$$
Figure 2. The Ratio of Early to Late Investment in Human Capital As a Function of the Skill Multiplier for Different Values of Complementarity

(Assumes $r = 0$)
When $\phi = 0$, the function (6) is

$$h' = m_2(h, \theta_1, l_1, l_2) = m_2(h, \theta_1, l_1^\gamma l_2^{1-\gamma}).$$
When CES complementarity is high, the skill multiplier $\gamma$ plays a limited role in shaping the ratio of early to late investments.

High early investments should be followed by high late investments.

As the degree of CES complementarity decreases, the role of the skill multiplier increases, and the higher the multiplier, the more investments should be concentrated in the early ages.
Alternative Market Environments

- In a complete-market model, optimal investment levels do not depend on the parental permanent shocks to wages or endowments or the parameters that characterize the utility function \( u(\cdot) \).
- Note, however, that even in this “perfect” credit market setting, parental investments depend on parental skills, \( h \), because these characteristics affect the returns to investment.
- From the point of view of the child, this is a market failure due to the accident of birth.
Consider the second credit constraint: the parental bequests must be non-negative and parents only have access to a risk-free bond, and not to contingent claims.

The problem of the parent is to maximize (8) subject to (7), the technology (6), and the liquidity constraint:

\[ b' \geq 0. \]  \hspace{1cm} (10)
If the constraint (10) binds, then early investments under lifetime liquidity constraints, $\hat{I}_1$, is lower than the early investment under the perfect credit market model, $I_1^*$. The same is true for the late investments: $\hat{I}_2 < I_2^*$. Under this formulation of market incompleteness, underinvestment in skills starts at early ages and continues throughout the life cycle of the child.
- Both early and late investments depend on parental initial wealth $b$ for the families for whom the constraint (10) binds.

- Children who come from constrained families with lower $b$ will have lower early and late investments.

- Interventions that occur at early stages would exhibit high returns, especially if they are followed up with resources to supplement late investments.
However, once the early stage investment is realized, late remediation for disadvantaged children would produce lower returns if early and late investments are not perfect substitutes and late investments are more productive the higher the level of early investments.

This explains fact five in section I. If complementarity and self-productivity are strong enough, this analysis also explains fact one. Skill gaps open up early and are perpetuated.
The effects of government policies on promoting the accumulation of human capital depend on the complementarity between early and late investments as well as on whether the policies were anticipated by parents or not.

For example, the short-run effects of an unanticipated policy that subsidizes late investments will have weaker effects the greater the complementarity between early and late investments.

If the technology is Leontief (5), there is no short-run impact of the policy on adolescent investment for children from disadvantaged environments.
There is, however, a long run effect of the policy.

If the policy is a permanent change announced before the child is born, the parents will adjust both early and late investments in response to the subsidy to late investments.

Note that the same is true for an exogenous increase in the returns to college education.

If there is strong complementarity between early and late investments, in the short-run we would expect weak reactions to the increase in returns as gauged by adolescent investment decisions for the children from very poor family backgrounds, but stronger reactions in the long run.
Figure D11a. College participation of high school graduates and GED holders (white males)
Figure 11b. College participation by race
Dependent high school graduates and GED holders
Males, ages eighteen to twenty-four
There is no trade-off between equity and efficiency in early childhood investments.

Government policies to promote early accumulation of human capital should be targeted to the children of poor families.

However, the optimal late intervention for a child from a disadvantaged environment depends critically on the nature of the technology.

If \( I_1 \) and \( I_2 \) are perfect CES complements, then a low level of \( I_1 \) cannot be compensated at any level of investment by a high \( I_2 \).

At a sufficiently high level of second-period investment, it is technically possible to offset low first period investment, but it may not be cost effective to do so.
The concepts of critical and sensitive periods are defined in terms of the technical possibilities of remediation.

Many noneconomists frame the question of remediation for adverse environments in terms of what is technically possible — not what is economically efficient.
Third constraint: parents are subject to lifetime liquidity constraints and constraints that prevent the parents from borrowing against their own future labor income, which may affect their ability to finance investments in the child’s early years.

To analyze this case, assume that parents productivity grows exogenously at rate $\alpha$. 

Let $s$ denote parental savings.

We write the constraints the parents face at each stage of the life cycle of the child as:

\[
c_1 + l_1 + \frac{s}{1 + r} = wh + b
\]

\[
c_2 + l_2 + \frac{b'}{1 + r} = w (1 + \alpha) h + s,
\]

where $s \geq 0$ and $b' \geq 0$. 
The restriction $s \geq 0$ says that parents cannot borrow income from their old age to finance consumption and investment when the child is in the first stage of the life cycle.

Some parents may be willing to do this, especially when $\alpha$ is high.

In the case when $s \geq 0$ and $b' \geq 0$ bind, and investments are not perfect substitutes, early income matters.
To see this, note that if \( u(c) = (c^\sigma - 1)/\sigma \) then, the ratio of early to late investment is

\[
\frac{l_1}{l_2} = \left[ \frac{\gamma}{(1 - \gamma)(1 + r)} \right]^{\frac{1}{1 - \phi}} \left[ \frac{(wh + b - l_1)}{\beta ((1 + \alpha)wh - l_2)} \right]^{\frac{1 - \sigma}{1 - \phi}}.
\]

If early income is low with respect to late income, the ratio \( l_1/l_2 \) will be lower than the optimal ratio.
The deviation from the optimal ratio will be larger the lower the elasticity of intertemporal substitution of consumption (captured by the parameter $\sigma$).

Early income would not matter if $\sigma = 1$, which would be the case when adult consumption at different stages of child development are perfect substitutes.

Substitutability through preferences can undo lack of substitutability in the technology of skill formation.
Our analysis of credit constrained families joined with a low value of $\phi$ interprets fact five of section I that the timing of family income in the early stages of childhood apparently affects the level of ability and achievement of the children, although there is still some controversy about the empirical importance of this effect.

It also interprets the evidence of Carneiro and Heckman and Cameron and Taber that conditioning on child ability, family income in the adolescent years has only a minor effect on adolescent schooling choices.
Cognitive and Noncognitive Skill Formation

- A large body of research documents the socioemotional basis of reason (see Damasio, 1994; LeDoux, 1996).
- Our analysis goes beyond this literature to consider how both cognitive and noncognitive skills emerge.
- We formalize a body of evidence that emotional skills promote learning.
- Direct developmental mechanisms relating cortisol to stress and the effects of cortisol on brain development have been documented by Suomi (1999) and Meaney (2001) for animals.
The framework previously developed readily accommodates skill vectors.

Let $\theta_t$ denote the vector of cognitive and noncognitive skills: $\theta_t = (\theta^C_t, \theta^N_t)$.

Let $I_t$ denote the vector of investment in cognitive and noncognitive skills: $I_t = (I^C_t, I^N_t)$.

We use $h = (h^C, h^N)$ to denote the parental cognitive and noncognitive skills.
Recursive technology for cognitive skills \((k = C)\), and noncognitive skills, \((k = N)\):

\[
\theta_{t+1}^k = f_t^k \left( \theta_t^C, \theta_t^N, I_t^k, h^C, h^N \right), \quad k \in \{C, N\}.
\] (11)

Technology (11) allows for cross-productivity effects: cognitive skills may affect the accumulation of noncognitive skills and vice versa.
If cognitive and/or noncognitive skills determine costs of effort, time preference or risk aversion parameters, parental investments serve to determine child and adult behavior.


Those authors build principal-agent models where parent (the principal) and child (the agent) agree on contracts in which parents financial transfers are conditional on observable measures of effort (e.g., earnings).

These contracts are designed so that the children are driven towards the level of effort desired by the parents.

In our model, parents directly shape child preferences.
Accounting for preference formation enables us to interpret the success of many early childhood programs targeted to disadvantaged children, which do not raise IQ, but which boost social performance.

This is point six of part I.

The controversy over Headstart fadeout in the 1960s may have been a consequence of looking only at cognitive measures.
The Perry Preschool Program had an IQ fadeout but a lasting effect on a variety of participants through age 40. They work harder, are less likely to commit crime and participate in many fewer social pathologies than do control group members.

Exact mechanism is unclear:

(a) Direct impact on noncognitive skills.
(b) An impact on IQ that fades but produces persistent effects on noncognitive skills.
Estimates of the Technology

- Cunha and Heckman (2006b) and Cunha, Heckman and Schennach (2006b) estimate recursive multistage technology (6) with cognitive and noncognitive skills where the outcomes produced by the skills are adult outcomes like schooling, earnings and occupational choice.

- They develop new econometric methods that extend factor analysis to a nonlinear setting.
They find strong evidence of self productivity and complementarity.

Their evidence is consistent with the literature demonstrating malleability of the prefrontal cortex governing executive function and socioeconomic development as well as the stability of IQ measures after age 10.
They find higher substitutability of early and late investment in producing noncognitive skills and lower substitutability of early investment in producing cognitive skills.

Higher stocks of noncognitive skills promote the self-productivity of cognitive skills; cognitive skill stocks promote the self productivity of noncognitive skills.

Higher levels of both cognitive and noncognitive skills raise the productivity of subsequent investment.
There is evidence of sensitive periods for parental investment.

The productivity of parental investment is higher in early stages for cognitive skills with a fall off in their productivity in later years.

The productivity of parental investment is higher at later stages for noncognitive skills.

This evidence is consistent with greater malleability of the prefrontal cortex governing socioemotional development into the early 20s, documented by Dahl (2004).
Cunha, Heckman and Schennach (2006b) estimate a strong interaction between initial endowments and parental investments that calls into question the conventional additive model of nature vs. nurture.

Even $\theta_1$, endowment at birth, is affected by environmental factors as a large literature documents.
Lessons for the Design of Policies

- Cunha and Heckman (2006b) simulate the nonlinear model of skill formation estimated by Cunha, Heckman and Schennach (2006b) to show the importance of self-productivity and complementarity for designing policies to reduce inequality arising from the accident of birth.

- We focus attention on children from disadvantaged backgrounds because at current levels of social inequality they benefit the most from policies that supplement early environments.
Consider three different policies.

The first policy is a Perry Preschool-like policy. It provides investments at early ages that move children from the first decile of child cognitive skills at entry age to the fourth decile of child skills at the age of exit from the program.

This gain can be achieved by moving parental investment from the bottom decile to around the 7th decile of the family investment distribution.

We assume that there is no follow-up investment.
We also consider a second policy for the same target population that postpones remediation until adolescence. It compensates early shortfalls by investing larger amounts in adolescent stages of the life cycle to produce approximately the same high school graduation rates that are observed in the Perry program.

To achieve Perry-like outcomes for this population solely through adolescent investment, it is necessary to move adolescent investment to the top of the parental investment distribution.

The present value of the costs of the investments in this adolescent remediation program is more than 35% larger than for the early intervention Perry Preschool program. Late remediation is possible but it is costly.
The case for early childhood interventions is based more on the importance of sensitive periods in the life cycle of the child than on the importance of critical periods.

We contrast early-only and late-only investment policies with a third policy that optimally distributes the resources spent in the second policy over the full life cycle of the child. A balanced investment strategy is the most efficient.
Table 2. Comparison of Different Investment Strategies

Disadvantaged Children: First Decile in the Distribution of Cognitive and Non-Cognitive Skills at Age 6

Mothers are in First Decile in the Distribution of Cognitive and Non-Cognitive Skills at Ages 14-21

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th>Changing initial conditions: moving children to the 4th decile of distribution of skills</th>
<th>Adolescent intervention: Moving investments at last transition from 1st to 9th decile</th>
<th>Changing initial conditions and performing a balanced intervention</th>
</tr>
</thead>
<tbody>
<tr>
<td>High School Graduation</td>
<td>0.4109</td>
<td>0.6579</td>
<td>0.6391</td>
<td>0.9135</td>
</tr>
<tr>
<td>Enrollment in College</td>
<td>0.0448</td>
<td>0.1264</td>
<td>0.1165</td>
<td>0.3755</td>
</tr>
<tr>
<td>Conviction</td>
<td>0.2276</td>
<td>0.1710</td>
<td>0.1733</td>
<td>0.1083</td>
</tr>
<tr>
<td>Probation</td>
<td>0.2152</td>
<td>0.1487</td>
<td>0.1562</td>
<td>0.0815</td>
</tr>
<tr>
<td>Welfare</td>
<td>0.1767</td>
<td>0.0905</td>
<td>0.0968</td>
<td>0.0259</td>
</tr>
</tbody>
</table>

Source: Cunha and Heckman (2006)
With our technology, we can rationalize the results found in the Perry program as an intervention that boosts parental investments (but not parental characteristics) from the first decile of investment in children to the seventh decile.
- The third column in Table 2 displays the performance of a 35% more costly policy that produces comparable educational outcomes for those obtained in the Perry-like intervention.

- Adolescent interventions can be effective, but they are more costly than early interventions.

- The greater cost associated with later remediation arises from lost gains in self-productivity and dynamic complementarity from early investments that are a key feature of our model.
• The empirical importance of dynamic complementarity generates an important insight for the design of policies.

• For a fixed expenditure, policies that are balanced increase returns and are more productive than policies tailored to one segment of the life cycle of the child.

• The returns to later investments are greater if high early investments are made.

• The intervention made later schooling more effective. If early interventions are followed up with later interventions in an optimal fashion, outcomes can be considerably improved.
The fourth column of Table 1 presents the results from a balanced policy.

The same amount of total investment distributed more evenly over the life cycle of the child produces more adult skills than a policy that concentrates attention on only one part of the child’s life cycle.
Summary and Extensions

- A technology of cognitive and noncognitive skill formation that features self productivity, dynamic complementarity and skill multipliers explains a variety of findings established in the child development and child intervention literatures.
We have shown the empirical limitations of the one period, perfect-substitutes-in-production model of child quality that has preoccupied the attention of theorists.

A multistage model explains a variety of empirical regularities in the literature.

Early investment plays a powerful role in shaping cognitive and noncognitive abilities.
Dynamic complementarity is empirically relevant.

Dynamic complementarity has a dual face:

(a) Early investments vital.

(b) Need to be followed up by later investments to be effective.

Later interventions are less effective if early investment is not made.
The technology of skill formation summarized in this paper can explain:

(1) Early emergence of skill gaps among socioeconomic groups.

(2) Critical and sensitive periods in animal and human species and low economic returns to most adolescent interventions.

(3) The effectiveness of early interventions targeted to children from disadvantaged families.
(4) Fadeout or weak effects in early investment programs not followed up by later investments.

(5) The unimportance of credit constraints for schooling during the child’s adolescent years, but the stronger evidence for such constraints in the early years.

(6) The importance of socioemotional skills in success in life and in fostering child development.
Although we have focused on cognitive and noncognitive skills, our analysis also applies to the formation of physical and mental health capital (see Cunha and Heckman, 2006c).
The evidence on the importance of early conditions on adult health (Case et al., 2005, and Barker, 1998) can be rationalized by our technology.

Stocks of cognitive and noncognitive skills facilitate the accumulation of health capital through self-regulation and choices.

Stocks of health skills also raise the productivity of schooling (Bhargava, 2008).