

Web Appendix for

“Formulating, Identifying and Estimating the
Technology of Cognitive and Noncognitive Skill
Formation”

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I Identification when η_t^N is serially correlated

In the discussion of identification in the text, we assumed that η_t^N is independently distributed over time. Consider the case in which η_t^N follows an AR(1) process:

$$\eta_{t+1}^N = \rho^N \eta_t^N + v_{t+1}^N,$$

where v_{t+1}^N has mean zero. Using the law of motion (16) from the paper, generalized to allow for time-varying coefficients, the difference $\theta_{t+1}^N - \rho^N \theta_t^N$ satisfies:

$$\begin{aligned} \theta_{t+1}^N - \rho \theta_t^N &= \gamma_{0,t+1}^N - \rho^N \gamma_{0,t}^N + \gamma_{1,t}^N \theta_t^N - \rho^N \gamma_{1,t-1}^N \theta_{t-1}^N + \gamma_{2,t}^N \theta_t^C - \rho^N \gamma_{2,t-1}^N \theta_{t-1}^C + \gamma_{3,t}^N I_t \\ &\quad - \rho^N \gamma_{3,t-1}^N I_{t-1} + v_{t+1}^N \end{aligned} \quad (\text{I.1})$$

or

$$\begin{aligned} \theta_{t+1}^N &= \gamma_{0,t+1}^N - \rho^N \gamma_{0,t}^N + (\rho^N + \gamma_{1,t}^N) \theta_t^N - \rho^N \gamma_{1,t-1}^N \theta_{t-1}^N + \gamma_{2,t}^N \theta_t^C - \rho^N \gamma_{2,t-1}^N \theta_{t-1}^C + \gamma_{3,t}^N I_t \\ &\quad - \rho^N \gamma_{3,t-1}^N I_{t-1} + v_{t+1}^N. \end{aligned} \quad (\text{I.2})$$

By the same logic

$$\begin{aligned} \theta_t^N &= \gamma_{0,t}^N - \rho^N \gamma_{0,t-1}^N + (\rho^N + \gamma_{1,t-1}^N) \theta_{t-1}^N - \rho^N \gamma_{1,t-2}^N \theta_{t-2}^N + \gamma_{2,t-1}^N \theta_{t-1}^C - \rho^N \gamma_{2,t-2}^N \theta_{t-2}^C \\ &\quad + \gamma_{3,t-1}^N I_t - \rho^N \gamma_{3,t-2}^N I_{t-2} + v_t^N. \end{aligned} \quad (\text{I.3})$$

Again, we use the measurement equations

$$Y_{1,t}^N, Y_{1,t-1}^N, Y_{1,t-1}^C, Y_{1,t-1}^I,$$

instead of

$$\theta_t^N, \theta_{t-1}^N, \theta_{t-1}^C, \theta_{t-1}^I.$$

As before, OLS regressions will produce inconsistent estimates, but we can use

$$(Y_{j,t-1}^N)_{j=2}^{m_t^N}, (Y_{j,t-1}^C)_{j=2}^{m_t^C} \text{ and } (Y_{j,t-1}^I)_{k=2}^{m_t^I}$$

as instruments for

$$Y_{1,t-1}^N, Y_{1,t-1}^C, \text{ or } Y_{t-1}^I$$

in (I.3). As a result, we can identify the parameters

$$\begin{aligned} &(\gamma_{0,t+1}^N - \rho^N \gamma_{0,t}^N), (\rho^N + \gamma_{1,t-1}^N), (\rho^N \gamma_{1,t-2}^N), (\gamma_{2,t-1}^N), (\rho^N \gamma_{2,t-2}^N), (\gamma_{3,t-1}^N), \\ &(\rho^N \gamma_{3,t-2}^N), (\gamma_{4,t-1}^N - \rho^N \gamma_{4,t-2}^N), \text{ and } (\gamma_{5,t-1}^N - \rho^N \gamma_{5,t-2}^N). \end{aligned}$$

If we apply our strategy to this setting and plug

$$Y_{1,t+1}^N, Y_{1,t}^N, Y_{1,t}^C, \text{ and } Y_{1,t}^I \text{ instead of } \theta_{t+1}^N, \theta_t^N, \theta_t^C, \text{ and } \theta_t^I$$

into (I.2) and use

$$(Y_{j,t}^N)_{j=2}^{m_t^N}, (Y_{j,t}^C)_{j=2}^{m_t^C}, \text{ and } (Y_{j,t}^I)_{k=2}^{m_t^I}$$

as instruments, we can obtain the parameters

$$(\gamma_{0,t+1}^N - \rho^N \gamma_{0,t}^N), (\rho^N + \gamma_{1,t}^N), (\rho^N \gamma_{1,t-1}^N), (\gamma_{2,t}^N), (\rho^N \gamma_{2,t-1}^N) \text{ and } (\gamma_{3,t}^N), (\rho^N \gamma_{3,t-1}^N).$$

We can recover ρ^N from two sources. First, from the estimation in (I.3) we identify $\gamma_{2,t-1}^N$, while in (I.2) we identify $\rho^N \gamma_{2,t-1}^N$. From the ratio of these two parameters we get ρ^N . Second, we can use the ratio of $\rho^N \gamma_{3,t-1}^N$ (which we obtain in our regression in (I.2)) to $\gamma_{3,t-1}^N$ (which we obtain from (I.3)). Once we identify ρ^N , we can recover the other parameters of interest. We can do a parallel analysis for the cognitive equation. This discussion assumes access to the relevant initial conditions of the process.

Web Appendix Table 1 Correlation - White Children NLSY/1979 - 6 years old

	Math	Reading	Composition	Antisocial	Anxious	Headstrong	Hyperactive	Conflict
Piat Math ¹	1.0000							
Piat Reading Recognition ¹	0.4794	1.0000						
Piat Composition ¹	0.4462	0.9196	1.0000					
Antisocial Score ¹	0.1298	0.0987	0.0662	1.0000				
Anxious Score ¹	0.1031	0.0834	0.0505	0.3952	1.0000			
Headstrong Score ¹	0.1255	0.0915	0.0619	0.5159	0.5587	1.0000		
Hyperactive Score ¹	0.1642	0.1645	0.1352	0.4989	0.4551	0.6019	1.0000	
Conflict Score ¹	0.0694	0.0617	0.0545	0.4877	0.4035	0.4196	0.3519	1.0000
Log Current Family Income ²	0.1665	0.1672	0.1427	0.0824	0.1038	0.1153	0.0734	0.1111
Number of Books ³	0.1696	0.2257	0.2346	0.0299	0.0993	0.1437	0.1788	0.0831
Musical Instrument ⁴	0.1673	0.1600	0.1401	0.1086	0.1325	0.0906	0.1445	0.1379
Newspaper ⁴	0.1248	0.1896	0.1786	0.1373	0.0609	0.0804	0.1163	0.1028
Child has special lessons ⁴	0.1407	0.1739	0.1406	0.1922	0.2061	0.1518	0.1981	0.1354
Child goes to museums ⁵	0.1158	0.0525	0.0106	0.1003	0.0732	0.0093	0.1329	0.0372
Child goes to theater ⁵	0.1470	0.1079	0.0860	0.0670	0.0740	0.0533	0.1035	0.0452
Education of the mother	0.2077	0.1880	0.2105	0.1436	0.1504	0.1002	0.2038	0.1482
Child ever sees father ⁴	0.0233	0.0626	0.0392	0.0010	0.0726	0.0798	0.0542	0.0234
Child spends time with father indoors ⁶	0.2009	0.1919	0.1773	0.1781	0.1397	0.1138	0.1795	0.1260
Child spends time with father outdoors ⁶	0.1275	0.0688	0.0779	0.2090	0.1030	0.1580	0.1936	0.1346
Child eats with father and mother ⁶	0.0981	0.0722	0.0620	0.1119	0.1085	0.0528	0.0292	0.0878
Child sees relatives and family friends ⁶	0.1091	0.0880	0.0560	-0.0031	0.0731	-0.0042	0.0424	0.0200
Mother's ASVAB (AR) ⁷	0.2434	0.2143	0.2060	0.1019	0.1013	0.1142	0.1506	0.0430
Mother's ASVAB (WK) ⁸	0.1947	0.2553	0.2743	0.1197	0.1492	0.0772	0.1686	0.1100
Mother's ASVAB (PC) ⁹	0.1695	0.2011	0.2044	0.1217	0.1541	0.0967	0.1911	0.0993
Mother's ASVAB (NO) ¹⁰	0.2273	0.1965	0.1933	0.1635	0.1395	0.1494	0.2005	0.1956
Mother's ASVAB (CS) ¹¹	0.2079	0.2321	0.2209	0.1882	0.1711	0.1523	0.1858	0.1887
Mother's ASVAB (MK) ¹²	0.2419	0.2259	0.2158	0.1152	0.1261	0.1368	0.1635	0.0718

¹The variables are standardized with mean zero and variance one across the entire CNLSY/79 sample.

²Family Income is inflation adjusted. Base year is 2000.

³The variable takes the value 1 if the child has no books, 2 if the child has 1 or 2 books, 3 if the child has 3 to 9 books and 4 if the child has 10 or more books.

⁴For example, for musical instrument, the variable takes value 1 if the child has a musical instrument at home and 0 otherwise. Other variables are defined accordingly.

⁵For example, for "museums", the variable takes the value 1 if the child never went to the museum in the last calendar year, 1 if the child went to the museum once or twice in the last calendar year, 3 if the child went to the museum several times in the past calendar year, 4 if the child went to the museum about once a month in the last calendar year, and 5 if the child went to a museum once a week in the last calendar year.

⁶For example, for "Child spends time with father indoors", the variable takes the value 1 if the child never spends time with the father indoors, 2 if the child spends time with the father indoors a few times in a year, 3 if the child spend time with the father indoors about once a month, 4 if the child spends time with the father indoors about once a week, 5 if the child spends time with the father indoors at least four times a week, and 6 if the child spends time with the father once a day or more often.

⁷AR stands for Arithmetic Reasoning. The variable is standardized with mean zero and variance one across the entire NLSY/1979 sample

⁸WK stands for Word Knowledge. The variable is standardized with mean zero and variance one across the entire NLSY/1979 sample

⁹PC stands for Paragraph Composition. The variable is standardized with mean zero and variance one across the entire NLSY/1979 sample

¹⁰NO stands for Numerical Operations. The variable is standardized with mean zero and variance one across the entire NLSY/1979 sample

¹¹CS stands for Coding Speed. The variable is standardized with mean zero and variance one across the entire NLSY/1979 sample

¹²MK stands for Mathematics Knowledge. The variable is standardized with mean zero and variance one across the entire NLSY/1979 sample

Web Appendix Table 2 Correlation - White Children NLSY/1979 - 7 years old

	Math	Reading	Composition	Antisocial	Anxious	Headstrong	Hyperactive	Conflict
Piat Math ¹	1.0000							
Piat Reading Recognition ¹	0.5945	1.0000						
Piat Composition ¹	0.5559	0.8520	1.0000					
Antisocial Score ¹	0.0950	0.1225	0.1201	1.0000				
Anxious Score ¹	0.0742	0.1246	0.1179	0.4395	1.0000			
Headstrong Score ¹	0.0533	0.1196	0.0926	0.5983	0.5173	1.0000		
Hyperactive Score ¹	0.2473	0.2585	0.1917	0.5035	0.4699	0.5505	1.0000	
Conflict Score ¹	0.0488	0.0801	0.0749	0.4680	0.4380	0.4157	0.3549	1.0000
Log Current Family Income ²	0.1387	0.1238	0.0920	0.0688	0.1693	0.0984	0.1427	0.0946
Number of Books ³	0.1655	0.0858	0.0669	0.1000	0.0432	0.0954	0.1113	0.0799
Musical Instrument ⁴	0.1371	0.0877	0.0851	0.1076	0.1062	0.1062	0.1198	0.0419
Newspaper ⁴	0.0796	0.0948	0.0762	0.1688	0.1075	0.1300	0.1167	0.1060
Child has special lessons ⁴	0.2393	0.1851	0.1419	0.2222	0.1644	0.1226	0.2437	0.1445
Child goes to museums ⁵	0.1646	0.1036	0.0230	0.1099	0.1270	0.1303	0.1550	0.0767
Child goes to theater ⁵	0.1194	0.0370	0.0043	0.0256	0.0440	0.0822	0.0867	-0.0004
Education of the mother	0.3174	0.2392	0.1943	0.1801	0.0981	0.1227	0.2604	0.1015
Child ever sees father ⁴	0.0448	0.0406	0.0113	-0.0118	0.0173	0.0102	-0.0315	-0.0200
Child spends time with father indoors ⁶	0.2060	0.1644	0.1654	0.1406	0.1178	0.1078	0.1967	0.0779
Child spends time with father outdoors ⁶	-0.0654	-0.0363	-0.0331	0.1857	0.1817	0.1678	0.1781	0.1419
Child eats with father and mother ⁶	0.0199	-0.0297	-0.0137	0.0839	0.1036	0.1237	0.0447	0.0003
Child sees relatives and family friends ⁶	-0.0430	-0.0046	0.0158	0.1435	0.1652	0.1234	0.0768	0.1250
Mother's ASVAB (AR) ⁷	0.3003	0.1929	0.1783	0.0326	-0.0350	0.0146	0.0989	0.0011
Mother's ASVAB (WK) ⁸	0.2923	0.2129	0.2120	0.0395	0.0380	0.0241	0.1451	-0.0260
Mother's ASVAB (PC) ⁹	0.3165	0.2087	0.2383	0.0123	0.0105	0.0149	0.1257	0.0486
Mother's ASVAB (NO) ¹⁰	0.1812	0.1281	0.1216	0.1090	0.0437	0.0869	0.1471	0.0621
Mother's ASVAB (CS) ¹¹	0.2288	0.1082	0.1122	0.1752	0.0717	0.1220	0.1776	0.1113
Mother's ASVAB (MK) ¹²	0.3079	0.2140	0.1727	0.0405	-0.0002	0.0447	0.1957	0.0215

¹The variables are standardized with mean zero and variance one across the entire CNLSY/79 sample.

²Family Income is inflation adjusted. Base year is 2000.

³The variable takes the value 1 if the child has no books, 2 if the child has 1 or 2 books, 3 if the child has 3 to 9 books and 4 if the child has 10 or more books.

⁴For example, for musical instrument, the variable takes value 1 if the child has a musical instrument at home and 0 otherwise. Other variables are defined accordingly.

⁵For example, for "museums", the variable takes the value 1 if the child never went to the museum in the last calendar year, 1 if the child went to the museum once or twice in the last calendar year, 3 if the child went to the museum several times in the past calendar year, 4 if the child went to the museum about once a month in the last calendar year, and 5 if the child went to a museum once a week in the last calendar year.

⁶For example, for "Child spends time with father indoors", the variable takes the value 1 if the child never spends time with the father indoors, 2 if the child spends time with the father indoors a few times in a year, 3 if the child spend time with the father indoors about once a month, 4 if the child spends time with the father indoors about once a week, 5 if the child spends time with the father indoors at least four times a week, and 6 if the child spends time with the father once a day or more often.

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¹⁰NO stands for Numerical Operations. The variable is standardized with mean zero and variance one across the entire NLSY/1979 sample

¹¹CS stands for Coding Speed. The variable is standardized with mean zero and variance one across the entire NLSY/1979 sample

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Web Appendix Table 3 Correlation - White Children NLSY/1979 - 8 years old

	Math	Reading	Composition	Antisocial	Anxious	Headstrong	Hyperactive	Conflict
Piat Math ¹	1.0000							
Piat Reading Recognition ¹	0.5331	1.0000						
Piat Composition ¹	0.5594	0.8184	1.0000					
Antisocial Score ¹	0.1263	0.1611	0.1858	1.0000				
Anxious Score ¹	0.0594	0.0693	0.0714	0.4138	1.0000			
Headstrong Score ¹	0.0806	0.0667	0.0798	0.5659	0.5901	1.0000		
Hyperactive Score ¹	0.1839	0.1691	0.2009	0.4865	0.4719	0.5671	1.0000	
Conflict Score ¹	0.0675	0.0751	0.1126	0.5341	0.3875	0.4074	0.2934	1.0000
Log Current Family Income ²	0.1661	0.1783	0.2035	0.1624	0.2024	0.1004	0.1626	0.0956
Number of Books ³	0.1492	0.0895	0.1000	0.1206	0.0768	0.0872	0.1202	0.0613
Musical Instrument ⁴	0.1371	0.1844	0.2112	0.0588	0.0580	0.0375	0.0285	0.0787
Newspaper ⁴	0.1710	0.1310	0.0966	0.1557	0.1542	0.0715	0.1933	0.0613
Child has special lessons ⁴	0.1881	0.2080	0.2187	0.1819	0.1709	0.1420	0.1619	0.1704
Child goes to museums ⁵	0.1712	0.1968	0.2327	0.0547	0.0072	-0.0076	0.0399	0.0131
Child goes to theater ⁵	0.1632	0.1291	0.1337	0.0369	0.0252	-0.0340	0.0244	0.0011
Education of the mother	0.1983	0.1489	0.1255	0.1559	0.1580	0.1137	0.2014	0.0773
Child ever sees father ⁴	-0.0565	-0.0004	-0.0090	-0.0223	0.0985	0.0145	0.0519	-0.0324
Child spends time with father indoors ⁶	0.1524	0.0964	0.1172	0.1724	0.2445	0.1595	0.2037	0.1615
Child spends time with father outdoors ⁶	0.0878	0.0338	0.0715	0.1510	0.2209	0.1442	0.1764	0.1631
Child eats with father and mother ⁶	0.0226	0.0699	0.0691	0.0544	0.0875	0.0432	0.0335	0.0855
Child sees relatives and family friends ⁶	-0.0040	0.0037	0.0027	0.0747	0.0509	0.0468	-0.0062	0.0488
Mother's ASVAB (AR) ⁷	0.3210	0.2532	0.2335	0.1460	0.1086	0.0849	0.1556	0.0247
Mother's ASVAB (WK) ⁸	0.2885	0.3076	0.2917	0.1530	0.0724	-0.0047	0.1237	0.0430
Mother's ASVAB (PC) ⁹	0.2806	0.2648	0.2582	0.1377	0.0617	0.0221	0.1121	0.0562
Mother's ASVAB (NO) ¹⁰	0.2986	0.2080	0.2191	0.1844	0.1595	0.1360	0.2116	0.1407
Mother's ASVAB (CS) ¹¹	0.3130	0.1908	0.1817	0.1802	0.1482	0.1462	0.1882	0.1271
Mother's ASVAB (MK) ¹²	0.2770	0.2187	0.1968	0.1479	0.1248	0.1187	0.1550	0.0345

¹The variables are standardized with mean zero and variance one across the entire CNLSY/79 sample.

²Family Income is inflation adjusted. Base year is 2000.

³The variable takes the value 1 if the child has no books, 2 if the child has 1 or 2 books, 3 if the child has 3 to 9 books and 4 if the child has 10 or more books.

⁴For example, for musical instrument, the variable takes value 1 if the child has a musical instrument at home and 0 otherwise. Other variables are defined accordingly.

⁵For example, for "museums", the variable takes the value 1 if the child never went to the museum in the last calendar year, 1 if the child went to the museum once or twice in the last calendar year, 3 if the child went to the museum several times in the past calendar year, 4 if the child went to the museum about once a month in the last calendar year, and 5 if the child went to a museum once a week in the last calendar year.

⁶For example, for "Child spends time with father indoors", the variable takes the value 1 if the child never spends time with the father indoors, 2 if the child spends time with the father indoors a few times in a year, 3 if the child spend time with the father indoors about once a month, 4 if the child spends time with the father indoors about once a week, 5 if the child spends time with the father indoors at least four times a week, and 6 if the child spends time with the father once a day or more often.

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Web Appendix Table 4 Correlation - White Children NLSY/1979 - 9 years old

	Math	Reading	Composition	Antisocial	Anxious	Headstrong	Hyperactive	Conflict
Piat Math ¹	1.0000							
Piat Reading Recognition ¹	0.5509	1.0000						
Piat Composition ¹	0.5717	0.7617	1.0000					
Antisocial Score ¹	0.1766	0.2741	0.2674	1.0000				
Anxious Score ¹	0.1371	0.1545	0.1253	0.4562	1.0000			
Headstrong Score ¹	0.1342	0.2094	0.1759	0.5726	0.6045	1.0000		
Hyperactive Score ¹	0.2372	0.3281	0.3356	0.5909	0.5119	0.5723	1.0000	
Conflict Score ¹	0.0828	0.1537	0.1207	0.5766	0.5341	0.4151	0.4291	1.0000
Log Current Family Income ²	0.1248	0.1220	0.1470	0.1299	0.1126	0.1026	0.1194	0.1159
Number of Books ³	0.1579	0.0988	0.1665	0.1266	0.1358	0.0881	0.1416	0.0933
Musical Instrument ⁴	0.1254	0.1507	0.1416	0.2009	0.1167	0.1324	0.1431	0.1673
Newspaper ⁴	0.1058	0.1828	0.1675	0.1525	0.0514	0.0906	0.0814	-0.0248
Child has special lessons ⁴	0.2916	0.2380	0.2788	0.2408	0.2029	0.2094	0.2453	0.1793
Child goes to museums ⁵	0.1555	0.0913	0.1489	0.1290	0.1025	0.1960	0.1026	-0.0087
Child goes to theater ⁵	0.2013	0.1752	0.1617	0.1366	0.1345	0.1991	0.1690	0.0341
Education of the mother	0.3391	0.2805	0.2777	0.2092	0.0891	0.1379	0.1770	0.0859
Child ever sees father ⁴	-0.0674	0.0311	0.0692	0.1102	0.0960	0.1053	0.0813	0.0067
Child spends time with father indoors ⁶	0.1352	0.1046	0.1230	0.2529	0.2574	0.2177	0.2276	0.1656
Child spends time with father outdoors ⁶	-0.0220	-0.0246	0.0531	0.2255	0.2713	0.2479	0.1631	0.1621
Child eats with father and mother ⁶	-0.0408	0.0174	0.0628	0.0946	0.1670	0.0924	0.0485	0.1380
Child sees relatives and family friends ⁶	0.0472	0.0961	0.0926	0.0604	0.0707	0.0437	0.0194	0.0956
Mother's ASVAB (AR) ⁷	0.3216	0.2575	0.2762	0.0749	0.0064	0.0653	0.0801	0.0149
Mother's ASVAB (WK) ⁸	0.3001	0.2744	0.3100	0.0567	0.0328	0.0458	0.1522	0.0262
Mother's ASVAB (PC) ⁹	0.2936	0.2597	0.2793	0.0601	0.0727	0.0733	0.1499	0.0503
Mother's ASVAB (NO) ¹⁰	0.3205	0.2603	0.2712	0.1282	0.0704	0.1329	0.1419	0.0773
Mother's ASVAB (CS) ¹¹	0.2447	0.2034	0.2328	0.1803	0.0794	0.1031	0.1336	0.1065
Mother's ASVAB (MK) ¹²	0.3580	0.3063	0.3171	0.1072	0.0342	0.0789	0.1262	0.0107

¹The variables are standardized with mean zero and variance one across the entire CNLSY/79 sample.

²Family Income is inflation adjusted. Base year is 2000.

³The variable takes the value 1 if the child has no books, 2 if the child has 1 or 2 books, 3 if the child has 3 to 9 books and 4 if the child has 10 or more books.

⁴For example, for musical instrument, the variable takes value 1 if the child has a musical instrument at home and 0 otherwise. Other variables are defined accordingly.

⁵For example, for "museums", the variable takes the value 1 if the child never went to the museum in the last calendar year, 1 if the child went to the museum once or twice in the last calendar year, 3 if the child went to the museum several times in the past calendar year, 4 if the child went to the museum about once a month in the last calendar year, and 5 if the child went to a museum once a week in the last calendar year.

⁶For example, for "Child spends time with father indoors", the variable takes the value 1 if the child never spends time with the father indoors, 2 if the child spends time with the father indoors a few times in a year, 3 if the child spend time with the father indoors about once a month, 4 if the child spends time with the father indoors about once a week, 5 if the child spends time with the father indoors at least four times a week, and 6 if the child spends time with the father once a day or more often.

⁷AR stands for Arithmetic Reasoning. The variable is standardized with mean zero and variance one across the entire NLSY/1979 sample

⁸WK stands for Word Knowledge. The variable is standardized with mean zero and variance one across the entire NLSY/1979 sample

⁹PC stands for Paragraph Composition. The variable is standardized with mean zero and variance one across the entire NLSY/1979 sample

¹⁰NO stands for Numerical Operations. The variable is standardized with mean zero and variance one across the entire NLSY/1979 sample

¹¹CS stands for Coding Speed. The variable is standardized with mean zero and variance one across the entire NLSY/1979 sample

¹²MK stands for Mathematics Knowledge. The variable is standardized with mean zero and variance one across the entire NLSY/1979 sample

Web Appendix Table 5 Correlation - White Children NLSY/1979 - 10 years old

	Math	Reading	Composition	Antisocial	Anxious	Headstrong	Hyperactive	Conflict
Piat Math ¹	1.0000							
Piat Reading Recognition ¹	0.5768	1.0000						
Piat Composition ¹	0.5252	0.7231	1.0000					
Antisocial Score ¹	0.1760	0.1988	0.1677	1.0000				
Anxious Score ¹	0.1428	0.1781	0.1189	0.5115	1.0000			
Headstrong Score ¹	0.1218	0.1099	0.0726	0.6013	0.6001	1.0000		
Hyperactive Score ¹	0.2547	0.3102	0.2115	0.5514	0.5585	0.5847	1.0000	
Conflict Score ¹	0.1127	0.1099	0.0507	0.5687	0.5577	0.4668	0.4814	1.0000
Log Current Family Income ²	0.1805	0.1820	0.1549	0.1921	0.1349	0.1288	0.1439	0.0588
Number of Books ³	0.1811	0.2307	0.2059	0.2514	0.1171	0.1328	0.1738	0.0579
Musical Instrument ⁴	0.1377	0.2177	0.1126	0.1880	0.1270	0.1133	0.1896	0.0996
Newspaper ⁴	0.2391	0.1615	0.1289	0.0603	0.0711	0.0743	0.0550	0.0843
Child has special lessons ⁴	0.2140	0.2642	0.2013	0.1884	0.1841	0.1520	0.1965	0.1621
Child goes to museums ⁵	0.1201	0.1197	0.1047	0.1284	0.0517	0.1549	0.0911	0.0418
Child goes to theater ⁵	0.1945	0.1759	0.1294	0.1306	0.0816	0.1507	0.1530	0.0581
Education of the mother	0.1871	0.2202	0.1759	0.2105	0.1798	0.1777	0.2456	0.0950
Child ever sees father ⁴	0.0519	0.0959	0.0666	0.0041	-0.0075	-0.0491	-0.0021	-0.0247
Child spends time with father indoors ⁶	0.1404	0.1539	0.1367	0.1589	0.1708	0.1681	0.1301	0.0861
Child spends time with father outdoors ⁶	0.0638	0.0570	-0.0090	0.0815	0.2176	0.1628	0.1226	0.0880
Child eats with father and mother ⁶	0.0065	0.0598	-0.0064	0.0285	0.1424	0.0272	0.1151	0.0951
Child sees relatives and family friends ⁶	-0.0058	-0.0018	-0.0568	0.0204	0.0355	-0.0107	0.0011	0.0192
Mother's ASVAB (AR) ⁷	0.2822	0.3062	0.2520	0.1316	0.1048	0.0837	0.1337	0.0464
Mother's ASVAB (WK) ⁸	0.2532	0.3474	0.2901	0.1434	0.0615	0.0426	0.1420	0.0140
Mother's ASVAB (PC) ⁹	0.2539	0.2866	0.2596	0.1500	0.0967	0.0787	0.1466	0.0708
Mother's ASVAB (NO) ¹⁰	0.2937	0.2781	0.1853	0.1947	0.1936	0.1215	0.2003	0.0811
Mother's ASVAB (CS) ¹¹	0.2376	0.2176	0.1566	0.1928	0.1536	0.1164	0.1613	0.0813
Mother's ASVAB (MK) ¹²	0.2493	0.2978	0.2301	0.1369	0.1320	0.1337	0.1694	0.0311

¹The variables are standardized with mean zero and variance one across the entire CNLSY/79 sample.

²Family Income is inflation adjusted. Base year is 2000.

³The variable takes the value 1 if the child has no books, 2 if the child has 1 or 2 books, 3 if the child has 3 to 9 books and 4 if the child has 10 or more books.

⁴For example, for musical instrument, the variable takes value 1 if the child has a musical instrument at home and 0 otherwise. Other variables are defined accordingly.

⁵For example, for "museums", the variable takes the value 1 if the child never went to the museum in the last calendar year, 1 if the child went to the museum once or twice in the last calendar year, 3 if the child went to the museum several times in the past calendar year, 4 if the child went to the museum about once a month in the last calendar year, and 5 if the child went to a museum once a week in the last calendar year.

⁶For example, for "Child spends time with father indoors", the variable takes the value 1 if the child never spends time with the father indoors, 2 if the child spends time with the father indoors a few times in a year, 3 if the child spend time with the father indoors about once a month, 4 if the child spends time with the father indoors about once a week, 5 if the child spends time with the father indoors at least four times a week, and 6 if the child spends time with the father once a day or more often.

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¹²MK stands for Mathematics Knowledge. The variable is standardized with mean zero and variance one across the entire NLSY/1979 sample

Web Appendix Table 6 Correlation - White Children NLSY/1979 - 11 years old

	Math	Reading	Composition	Antisocial	Anxious	Headstrong	Hyperactive	Conflict
Piat Math ¹	1.0000							
Piat Reading Recognition ¹	0.5730	1.0000						
Piat Composition ¹	0.5274	0.7072	1.0000					
Antisocial Score ¹	0.1584	0.2207	0.1969	1.0000				
Anxious Score ¹	0.1198	0.1274	0.1555	0.5024	1.0000			
Headstrong Score ¹	0.2100	0.1894	0.2169	0.5840	0.6404	1.0000		
Hyperactive Score ¹	0.2625	0.3016	0.3329	0.5533	0.5591	0.5993	1.0000	
Conflict Score ¹	0.1277	0.1532	0.2157	0.5385	0.5331	0.4485	0.5011	1.0000
Log Current Family Income ²	0.2795	0.2286	0.2020	0.2510	0.1359	0.2184	0.2163	0.2350
Number of Books ³	0.1866	0.1971	0.1598	0.2279	0.1052	0.0894	0.2199	0.1157
Musical Instrument ⁴	0.2165	0.2153	0.1784	0.2797	0.1672	0.1592	0.1823	0.1590
Newspaper ⁴	0.1100	0.2107	0.2779	0.1427	0.0683	0.0910	0.1620	0.0773
Child has special lessons ⁴	0.2388	0.2432	0.2366	0.3210	0.1708	0.1574	0.3113	0.2683
Child goes to museums ⁵	0.2058	0.1491	0.1714	0.0729	0.0630	0.0984	0.1657	0.0575
Child goes to theater ⁵	0.2006	0.1804	0.1679	0.0613	0.0761	0.0923	0.1403	0.0744
Education of the mother	0.3964	0.3368	0.3067	0.2044	0.0383	0.1525	0.1804	0.0965
Child ever sees father ⁴	-0.0466	0.0443	-0.0270	-0.0668	-0.0749	-0.0981	-0.0618	-0.0270
Child spends time with father indoors ⁶	0.1004	0.0676	0.0899	0.2017	0.1717	0.1387	0.1397	0.1032
Child spends time with father outdoors ⁶	0.0193	-0.0395	-0.0030	0.1820	0.2129	0.1839	0.1746	0.1578
Child eats with father and mother ⁶	-0.0712	-0.0130	0.0624	0.2238	0.2424	0.1908	0.1474	0.1239
Child sees relatives and family friends ⁶	0.0663	0.0540	0.1300	0.0418	0.1619	0.0902	0.0606	0.1284
Mother's ASVAB (AR) ⁷	0.3469	0.2828	0.3194	0.0664	-0.0417	0.0640	0.1206	0.0320
Mother's ASVAB (WK) ⁸	0.3540	0.3599	0.3298	0.1040	-0.0125	0.0518	0.1338	0.0332
Mother's ASVAB (PC) ⁹	0.3001	0.3330	0.3201	0.1060	0.0179	0.0721	0.1706	0.0953
Mother's ASVAB (NO) ¹⁰	0.2862	0.2666	0.3021	0.1271	0.0709	0.1047	0.1493	0.0929
Mother's ASVAB (CS) ¹¹	0.2797	0.1856	0.2854	0.1355	0.1349	0.1635	0.2048	0.1766
Mother's ASVAB (MK) ¹²	0.3906	0.3095	0.3021	0.0853	-0.0331	0.0994	0.1697	0.0723

¹The variables are standardized with mean zero and variance one across the entire CNLSY/79 sample.

²Family Income is inflation adjusted. Base year is 2000.

³The variable takes the value 1 if the child has no books, 2 if the child has 1 or 2 books, 3 if the child has 3 to 9 books and 4 if the child has 10 or more books.

⁴For example, for musical instrument, the variable takes value 1 if the child has a musical instrument at home and 0 otherwise. Other variables are defined accordingly.

⁵For example, for "museums", the variable takes the value 1 if the child never went to the museum in the last calendar year, 1 if the child went to the museum once or twice in the last calendar year, 3 if the child went to the museum several times in the past calendar year, 4 if the child went to the museum about once a month in the last calendar year, and 5 if the child went to a museum once a week in the last calendar year.

⁶For example, for "Child spends time with father indoors", the variable takes the value 1 if the child never spends time with the father indoors, 2 if the child spends time with the father indoors a few times in a year, 3 if the child spend time with the father indoors about once a month, 4 if the child spends time with the father indoors about once a week, 5 if the child spends time with the father indoors at least four times a week, and 6 if the child spends time with the father once a day or more often.

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⁹PC stands for Paragraph Composition. The variable is standardized with mean zero and variance one across the entire NLSY/1979 sample

¹⁰NO stands for Numerical Operations. The variable is standardized with mean zero and variance one across the entire NLSY/1979 sample

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¹²MK stands for Mathematics Knowledge. The variable is standardized with mean zero and variance one across the entire NLSY/1979 sample

Web Appendix Table 7 Correlation - White Children NLSY/1979 - 12 years old

	Math	Reading	Composition	Antisocial	Anxious	Headstrong	Hyperactive	Conflict
Piat Math ¹	1.0000							
Piat Reading Recognition ¹	0.5845	1.0000						
Piat Composition ¹	0.5589	0.7124	1.0000					
Antisocial Score ¹	0.2208	0.1676	0.1462	1.0000				
Anxious Score ¹	0.1545	0.1038	0.0974	0.5052	1.0000			
Headstrong Score ¹	0.2043	0.1425	0.1563	0.6245	0.6237	1.0000		
Hyperactive Score ¹	0.2417	0.2079	0.2018	0.5507	0.5327	0.6028	1.0000	
Conflict Score ¹	0.1317	0.1479	0.0406	0.5437	0.5431	0.5096	0.4343	1.0000
Log Current Family Income ²	0.2333	0.2375	0.2230	0.0985	0.2186	0.1528	0.1198	0.0980
Number of Books ³	0.1048	0.1381	0.1836	0.0904	0.0104	0.0557	0.0347	0.0869
Musical Instrument ⁴	0.1811	0.1595	0.2010	0.1080	0.0877	0.1605	0.1317	0.0567
Newspaper ⁴	0.2102	0.1950	0.1966	0.1225	0.0768	0.1221	0.1320	0.1370
Child has special lessons ⁴	0.2138	0.1501	0.1810	0.1552	0.1308	0.2132	0.1650	0.1241
Child goes to museums ⁵	0.1141	0.1506	0.1493	0.1057	0.1042	0.1044	0.0809	0.0515
Child goes to theater ⁵	0.1190	0.1562	0.1584	0.1646	0.0955	0.1489	0.1146	0.1202
Education of the mother	0.2727	0.2461	0.2987	0.1444	0.2113	0.2074	0.2148	0.1353
Child ever sees father ⁴	0.1127	0.1259	0.0836	0.0095	0.0477	0.0505	0.0153	0.0468
Child spends time with father indoors ⁶	0.1222	0.0992	0.1426	0.1223	0.1100	0.1370	0.0939	0.0866
Child spends time with father outdoors ⁶	0.1129	-0.0056	-0.0054	0.1423	0.1893	0.1991	0.0916	0.1503
Child eats with father and mother ⁶	0.0449	0.0525	0.0394	0.1063	0.0982	0.1089	0.0420	0.0416
Child sees relatives and family friends ⁶	0.0247	0.0377	0.0152	-0.0399	0.0216	-0.0717	-0.0310	-0.0495
Mother's ASVAB (AR) ⁷	0.3050	0.3451	0.4212	0.0359	0.0764	0.0672	0.1041	0.0131
Mother's ASVAB (WK) ⁸	0.2341	0.3968	0.4136	0.0101	0.0564	0.0445	0.1191	0.0125
Mother's ASVAB (PC) ⁹	0.2238	0.3221	0.3427	0.0581	0.1095	0.0590	0.1583	0.0845
Mother's ASVAB (NO) ¹⁰	0.3528	0.3356	0.3277	0.1253	0.1540	0.1645	0.1827	0.1268
Mother's ASVAB (CS) ¹¹	0.2685	0.2748	0.2679	0.1164	0.1051	0.1489	0.1818	0.1274
Mother's ASVAB (MK) ¹²	0.3163	0.3292	0.4155	0.0385	0.0890	0.0827	0.0901	-0.0225

¹The variables are standardized with mean zero and variance one across the entire CNLSY/79 sample.

²Family Income is inflation adjusted. Base year is 2000.

³The variable takes the value 1 if the child has no books, 2 if the child has 1 or 2 books, 3 if the child has 3 to 9 books and 4 if the child has 10 or more books.

⁴For example, for musical instrument, the variable takes value 1 if the child has a musical instrument at home and 0 otherwise. Other variables are defined accordingly.

⁵For example, for "museums", the variable takes the value 1 if the child never went to the museum in the last calendar year, 1 if the child went to the museum once or twice in the last calendar year, 3 if the child went to the museum several times in the past calendar year, 4 if the child went to the museum about once a month in the last calendar year, and 5 if the child went to a museum once a week in the last calendar year.

⁶For example, for "Child spends time with father indoors", the variable takes the value 1 if the child never spends time with the father indoors, 2 if the child spends time with the father indoors a few times in a year, 3 if the child spend time with the father indoors about once a month, 4 if the child spends time with the father indoors about once a week, 5 if the child spends time with the father indoors at least four times a week, and 6 if the child spends time with the father once a day or more often.

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¹²MK stands for Mathematics Knowledge. The variable is standardized with mean zero and variance one across the entire NLSY/1979 sample

Web Appendix Table 8 Correlation - White Children NLSY/1979 - 13 years old

	Math	Reading	Composition	Antisocial	Anxious	Headstrong	Hyperactive	Conflict
Piat Math ¹	1.0000							
Piat Reading Recognition ¹	0.5609	1.0000						
Piat Composition ¹	0.5174	0.6983	1.0000					
Antisocial Score ¹	0.2014	0.2850	0.2237	1.0000				
Anxious Score ¹	0.1455	0.1406	0.1100	0.5496	1.0000			
Headstrong Score ¹	0.1240	0.1843	0.0927	0.6576	0.5809	1.0000		
Hyperactive Score ¹	0.2222	0.2987	0.2436	0.5119	0.5230	0.5603	1.0000	
Conflict Score ¹	0.0490	0.1676	0.0972	0.4706	0.5743	0.4183	0.4352	1.0000
Log Current Family Income ²	0.1192	0.1321	0.1503	0.1255	0.1192	0.1161	0.0905	0.1170
Number of Books ³	0.1989	0.3007	0.2962	0.2656	0.1006	0.1398	0.1719	0.1366
Musical Instrument ⁴	0.1679	0.1946	0.2182	0.1299	0.0976	0.0794	0.1005	0.1042
Newspaper ⁴	0.1328	0.2371	0.2227	0.1385	0.0690	0.1404	0.1122	0.1043
Child has special lessons ⁴	0.1616	0.1277	0.1484	0.2653	0.2164	0.2116	0.2085	0.1582
Child goes to museums ⁵	0.0189	0.1361	0.1338	0.2278	0.1048	0.1989	0.0968	0.1060
Child goes to theater ⁵	0.0517	0.1281	0.1000	0.1403	0.0624	0.2043	0.0975	0.0960
Education of the mother	0.3412	0.3340	0.3333	0.2207	0.0944	0.1367	0.1458	0.1372
Child ever sees father ⁴	-0.0463	-0.0205	0.0625	0.0412	-0.0272	0.0242	-0.0226	-0.0830
Child spends time with father indoors ⁶	-0.0327	0.0621	0.0847	0.1668	0.1021	0.1080	0.0767	0.0412
Child spends time with father outdoors ⁶	-0.0889	0.0205	0.0181	0.1458	0.1712	0.1780	0.1418	0.0826
Child eats with father and mother ⁶	0.0054	0.0353	0.0272	0.0957	0.0531	0.1135	0.0117	0.0265
Child sees relatives and family friends ⁶	-0.0514	0.0147	0.0097	0.0641	0.0603	0.0990	-0.0019	0.0786
Mother's ASVAB (AR) ⁷	0.3014	0.3600	0.3306	0.0974	-0.0075	0.0359	0.1117	0.0062
Mother's ASVAB (WK) ⁸	0.2439	0.3923	0.3520	0.0785	0.0059	0.0041	0.0188	0.0863
Mother's ASVAB (PC) ⁹	0.2573	0.3525	0.2754	0.0646	-0.0010	-0.0082	0.0562	0.0774
Mother's ASVAB (NO) ¹⁰	0.2509	0.3026	0.2368	0.1847	0.0435	0.0792	0.1268	0.1396
Mother's ASVAB (CS) ¹¹	0.2686	0.2988	0.2761	0.1859	0.0538	0.1332	0.2468	0.1636
Mother's ASVAB (MK) ¹²	0.3238	0.3806	0.3122	0.1397	0.0353	0.1000	0.1433	0.0736

¹The variables are standardized with mean zero and variance one across the entire CNLSY/79 sample.

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Web Appendix Table 9				
The Technology Equations¹				
Robustness Check: Normalize the Factor in Log Family Income				
We use Adult Earnings data to anchor the metric of the factor				
Estimated Parameter Values - Technology from Period 1 to Period 2				
	Next Period Noncognitive Skills		Next Period Cognitive Skills	
	Mean	Standard Error	Mean	Standard Error
Current Period Noncognitive Skills	0.9850	0.0135	0.0605	0.0222
Current Period Cognitive Skills	0.0513	0.0609	0.9197	0.0821
Current Period Investment	0.0544	0.0110	0.1126	0.0143
Mother's Education	0.0029	0.0036	-0.0050	0.0050
Mother's Ability	0.0010	0.0991	0.0506	0.0719
Variance of Shocks	0.1424	0.0158	0.0653	0.0161
Estimated Parameter Values - Technology from Period 2 to Period 3				
	Next Period Noncognitive Skills		Next Period Cognitive Skills	
	Mean	Standard Error	Mean	Standard Error
Current Period Noncognitive Skills	0.9384	0.0135	0.0212	0.0079
Current Period Cognitive Skills	-0.0449	0.0808	0.8844	0.0233
Current Period Investment	0.1062	0.0190	0.0364	0.0146
Mother's Education	-0.0026	0.0073	0.0131	0.0086
Mother's Ability	-0.0078	0.0148	0.0045	0.0067
Variance of Shocks	0.1284	0.0169	0.0231	0.0065
Estimated Parameter Values - Technology from Period 3 to Period 4				
	Next Period Noncognitive Skills		Next Period Cognitive Skills	
	Mean	Standard Error	Mean	Standard Error
Current Period Noncognitive Skills	0.7570	0.0318	0.0014	0.0031
Current Period Cognitive Skills	0.0418	0.0277	0.9099	0.0412
Current Period Investment	0.0450	0.0117	0.0380	0.0129
Mother's Education	0.0141	0.0122	0.0021	0.0013
Mother's Ability	-0.0135	0.0186	0.0194	0.0311
Variance of Shocks	0.1568	0.0183	0.0104	0.0021

¹Let $Y_t^N = (Y_{1,t}^N, \dots, Y_{m_t^N,t}^N)'$ denote the measurements of noncognitive skills. Let $Y_t^C = (Y_{1,t}^C, \dots, Y_{m_t^C,t}^C)'$ denote the measurements of cognitive skills. Let $X_t = (X_{1,t}, \dots, X_{m_t^I,t})'$ denote the measurements of parental investment (from the HOME-SF score). Let $Y_t = (Y_t^N, Y_t^C, X_t)$. Let $\theta = (\theta_t^N, \theta_t^C, I_t)$ denote the noncognitive, cognitive and investment dynamic factors, respectively. Let S denote mother's education and A denote mother's cognitive ability. The measurement equations are:

$$Y_t = \alpha_t \theta_t + \varepsilon_t$$

and the technology equations are:

$$\theta_{t+1} = \gamma_t \theta_t + \psi_{1,t} S + \psi_{2,t} A + \eta_{t+1}$$

where α_t is the factor-loading matrix, γ_t is the technology-parameters matrix, $\psi_{k,t}$ are parameter vectors. The vectors $\varepsilon_t, \eta_{t+1}$ contain the uniquenesses of the measurement equations and the error terms in the technology equations. In table 5.1 we show the estimated parameter values and standard errors of γ_t , $\psi_{1,t}$, and $\psi_{2,t}$ as well as the $Var(\eta_{t+1}^N)$ and $Var(\eta_{t+1}^C)$.

Web Appendix Table 10				
The Technology Equations¹				
Robustness Check: We do not use Log Family Income and Normalize the Factor in "Number of We use Earnings data to anchor the metric of the factor"				
Estimated Parameter Values - Technology from Period 1 to Period 2				
	Next Period Noncognitive Skills		Next Period Cognitive Skills	
	Mean	Standard Error	Mean	Standard Error
Current Period Noncognitive Skills	0.9863	0.0123	0.0631	0.0233
Current Period Cognitive Skills	0.0524	0.0621	0.9251	0.0908
Current Period Investment	0.1752	0.0342	0.4359	0.0932
Mother's Education	0.0033	0.0032	-0.0082	0.0069
Mother's Ability	0.0070	0.0932	0.0567	0.0653
Variance of Shocks	0.1426	0.0159	0.0646	0.0122
Estimated Parameter Values - Technology from Period 2 to Period 3				
	Next Period Noncognitive Skills		Next Period Cognitive Skills	
	Mean	Standard Error	Mean	Standard Error
Current Period Noncognitive Skills	0.9397	0.0210	0.0216	0.0061
Current Period Cognitive Skills	-0.0438	0.0655	0.8869	0.0421
Current Period Investment	0.4353	0.0612	0.1518	0.0229
Mother's Education	-0.0031	0.0051	0.0123	0.0068
Mother's Ability	-0.0054	0.0129	0.0065	0.0054
Variance of Shocks	0.1284	0.0154	0.0232	0.0071
Estimated Parameter Values - Technology from Period 3 to Period 4				
	Next Period Noncognitive Skills		Next Period Cognitive Skills	
	Mean	Standard Error	Mean	Standard Error
Current Period Noncognitive Skills	0.7599	0.0322	0.0019	0.0044
Current Period Cognitive Skills	0.0521	0.0241	0.9116	0.0499
Current Period Investment	0.0567	0.0184	0.0507	0.0192
Mother's Education	0.0083	0.0131	0.0036	0.0021
Mother's Ability	-0.0069	0.0154	0.0193	0.0233
Variance of Shocks	0.1574	0.0179	0.0103	0.0020

¹Let $Y_t^N = (Y_{1,t}^N, \dots, Y_{m_t^N,t}^N)'$ denote the measurements of noncognitive skills. Let $Y_t^C = (Y_{1,t}^C, \dots, Y_{m_t^C,t}^C)'$ denote the measurements of cognitive skills. Let $X_t = (X_{1,t}, \dots, X_{m_t^I,t})'$ denote the measurements of parental investment (from the HOME-SF score). Let $Y_t = (Y_t^N, Y_t^C, X_t)$. Let $\theta = (\theta_t^N, \theta_t^C, I_t)$ denote the noncognitive, cognitive and investment dynamic factors, respectively. Let S denote mother's education and A denote mother's cognitive ability. The measurement equations are:

$$Y_t = \alpha_t \theta_t + \varepsilon_t$$

and the technology equations are:

$$\theta_{t+1} = \gamma_t \theta_t + \psi_{1,t} S + \psi_{2,t} A + \eta_{t+1}$$

where α_t is the factor-loading matrix, γ_t is the technology-parameters matrix, $\psi_{k,t}$ are parameter vectors. The vectors $\varepsilon_t, \eta_{t+1}$ contain the uniquenesses of the measurement equations and the error terms in the technology equations. In table 5.4 we perform robustness tests for the existence of sensitive periods for cognitive and noncognitive skill developments. In particular, we want to investigate whether the results from table 5.1 are due to the choice of normalization in factor loadings for investment. We show that although the parameter values change, the ordering of the effects is the same as in table 5.1.