Notes on “The Importance of Bundling in a Gorman-Lancaster Model of Earnings”

Heckman and Scheinkman,
*Review of Economic Studies* 54(2), 1987

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Econ 345
This draft, January 16, 2007
• One version of an “hedonic model.”
• There are \( n \) sectors.
• Output in each sector depends on aggregate amounts of skill in each sector.
• \( x \) is an \( m \)-dimensional vector of skills.
• \( \psi_n \) is a vector of skill prices in sector \( n \).
• \( \overline{X}_n \) is aggregate skill vector.
• With uniform pricing of skills, there is no sectoral choice issue for workers (all sectors look alike).
Structural Wage Equations

Skill prices

\[ w_n = \frac{\partial F^n (X_n)}{\partial X_n} \]

Worker \( i \); Sector \( n \)

Earnings \( Y_{in} \) of worker \( i \) at sector \( n \):

\[ Y_{in} = \bar{x} \cdot \bar{w}_n, \]  \hspace{1cm} (1)

\[ n = 1, \ldots, N, \quad i = 1, \ldots, I \]

Allow for measurement error \( \varepsilon_{in} \):

\[ Y_{in} = \bar{x} \cdot \bar{w}_n + \varepsilon_{in}, \]  \hspace{1cm} (2)

\[ n = 1, \ldots, N, \quad i = 1, \ldots, I \]
Conditions for Uniform Pricing of Characteristics in a Multisector Economy

The set of workers is assumed to be given by a closed ball $T$ in some euclidean space $R^P$. Each worker $t \in T$ possess an m-vector of endowments

$$\tilde{x}(t) = (x_1(t), \ldots, x_m(t)).$$

We assume that the functions $x_i$ are bounded, non-negative and measurable.
There are $n$ sectors. In sector $j$, $\ell_j$ identical firms produce a single good according to a production function $f_j(a)$ where $\hat{a}_j$ is the total vector of skill $i$ used by the firm.

Workers must sell all their endowment to a single firm. This bundling of skills is the distinctive aspect of our problem. If a firm in sector $j$ hires workers in a subset $C$ of $T$, it produces $f_j(\hat{a}_j)$ where

$$\hat{a}_j = \int_C \hat{x}(t) \, dt.$$
An allocation is a partition of $T$ into disjoint measurable subsets $C^k_j$ where $k = 1, \ldots, \ell_j$, $j = 1, \ldots, n$, i.e., $C^k_j$ denotes the set of workers assigned to the $k$-th firm of sector $j$. Given an allocation we write

$$C_j = \bigcup_{k=1}^{\ell_j} C^k_j,$$

i.e., the set of all workers assigned to sector $j$. 
An allocation is a *competitive equilibrium* if there exists an \( m \times n \) matrix \((w_{ij})\), \( i = 1, \ldots, m \), \( j = 1, \ldots, n \) of skill prices in sector such that

(a) For each firm \( k \) in sector \( j \), \( a_j^k = \int_{C_j^k} \tilde{x}(t) \, dt \) solves

\[
\max p_j f_j(a_j^k) - \langle w_j, a_j^k \rangle
\]

subject to \( a_j^k = \int_{C_j^k} \tilde{x}(t) \, dt \), where \( C_j^k \) is a measurable subset of \( T \).

(b) For almost all \( t \in T \), if \( t \in C_j \) then

\[
\langle w_j, \tilde{x}(t) \rangle \geq \langle w_{\ell}, \tilde{x}(t) \rangle, \quad \ell = 1, 2, \ldots, n.
\]
Figure 1

Contract Curve

Feasible Region

Equilibrium Point

(0,0)
Figure 2

Contract Curve

Feasible Region

Equilibrium Point

(0,0)

a_1

a_2
Figure 3

Equilibrium Point

Contract Curve

Feasible Region
Characterizing the Optimum

A matrix \( \hat{a}_{j}^{k}, j = 1, \ldots, n, k = 1, \ldots, \ell_j \) is called a feasible state if there exists an allocation \( C_{j}^{k} \) with

\[
\int \hat{x}(t) \, dt = \hat{a}_{j}^{k}.
\]

For any feasible state \( \hat{a}_{j}^{k} \) we define

\[
a_j = \sum_{k=1}^{\ell_j} \hat{a}_{j}^{k}
\]

as the total skill vector in industry \( j \).
Lemma (1)

If $\underline{a}_j^k$ is feasible, so is

$$\underline{a}_j^k = \ell_j^{-1} \underline{a}_j, \ k = 1, \ldots, \ell_j, \ j = 1, \ldots, n.$$ 

We are interested in feasible industry states $\underline{a} = (\underline{a}_1, \ldots, \underline{a}_n)$ where each $\underline{a}_j \in R^m$ corresponds to a skill vector allocated to industry $j$. In order for $\underline{a}$ to be feasible there must exist measurable sets $C_j, j = 1, \ldots, n$, mutually disjoint with

$$\bigcup_{j=1}^n C_j = T.$$
Define

\[ K = \{ (a_1, \ldots, a_n) \mid a_j \in \mathbb{R}^m, a_j = \int_{C_j} \bar{x}(t) \, dt, \]  

\[ \bigcup_{j=1}^{n} C_j = T, \quad C_j, C_{j'} = \text{if } j \neq j' \}. \]
Lemma (2)

$K$ is convex.

$$
\max \sum_{j=1}^{n} p_j \ell_j f_j \left( a_j / \ell_j \right)
$$

such that

$$a = (a_1, \ldots, a_n) \in K, \quad K \text{ a convex set.}$$

Notice that (3) is a convex problem. If $a$ solves (3) and $a' \in K$, then, $\lambda a + (1 - \lambda)a' \in K$ and

$$h(\lambda) = \sum_{j=1}^{n} p_j \ell_j f_j \left( \ell_j^{-1} \left( \lambda a_j + (1 - \lambda)a'_j \right) \right)$$

satisfies $h'(1) \geq 0$. 
Thus

\[ \sum_{j=1}^{n} p_j \left\langle \frac{\partial f_j}{\partial \ell} (\bar{\alpha} / \ell_j), \bar{\alpha} - \bar{\alpha}' \right\rangle \geq 0. \]  

(4)

As a consequence of (3) and (4) we have the following theorem.
Theorem

If $\bar{a} \in \text{Int}(K)$ then (4) reduces to the usual equality of marginal products, i.e.,

$$p_j \frac{\partial f_j}{\partial a} (\bar{a}_j/\ell_j) = p_k \frac{\partial f_k}{\partial a} (\bar{a}_k/\ell_k)$$

for each $j, k$, and such an optimum may be decentralized by setting

$$w_j = p_1 \frac{\partial f_1}{\partial a} (\bar{a}_1/\ell_1)$$

for all $j$. ■
When \( m = n = 2 \), so there are two sectors and two characteristics, the endowment of the economy is described by a set

\[
E = \{ a \in R^2 \mid (a, \int_T x(t) \, dt - a) \in K \}.
\]

\( E \) is obviously analogous to the Edgeworth Box Diagram.
Example 1

Let $T = [0, 1], m = n = 2$ and $x_i(t) = 2t, i = 1, 2$. Let 
$f_j(a_1, a_2) = (a_j)^{\alpha}, j = 1, 2, 0 < \alpha < 1$. Here 
$E\{(t, t) \in R^2 \mid 0 \leq t \leq 1\}$. An optimal $\bar{a} \in E$. Clearly 
$0 < \bar{a}_i < 1$ for positive prices and the constraint is binding. 
Furthermore, since for any $0 \leq t \leq 1$, $(t, t)$ is feasible, and 

$$
\frac{\partial f_1(\bar{a}/\ell_1)}{\partial a_2} = \frac{\partial f_2}{\partial a_1}(((1, 1) - \bar{a})/\ell_2) = 0,
$$

from (4), it follows that 

$$
p_1 \frac{\partial f_1(\bar{a}/\ell_1)}{\partial a_1} = p_2 \frac{\partial f_2(((1, 1) - \bar{a})/\ell_2)}{\partial a_2}.
$$
Example 1 (cont.)

Let
\[ w_{jj} = \frac{1}{2} p_1 \frac{\partial f_1(\bar{a}/\ell_1)}{\partial a_1} \quad \text{for} \quad j = 1, 2. \]

At such wages, firms maximize profits taking into consideration that they must hire both skills in fixed propositions. In fact, any skill price \( w_1 \) and \( w_2 \) such that
\[ w_1 + w_2 + p_1 \frac{\partial f_1}{\partial a_1}(\bar{a}/\ell_1) \] will yield a competitive equilibrium.
Example 2

Let $T = [0, 1]$. Further let $n = m = 2$. Let $\bar{x}(t) = (1, 1)$ if $0 \leq t \leq 1/2$, $\bar{x}(t) = 1/2$ if $t > 1/2$. The set $E$ is a parallelogram in $\mathbb{R}^2$ with vertices $(0, 0); (1/2, 1/2); (1/2); (1/2, 1)$ and $(1, 3/2)$. 
Suppose that a constrained optimum is attained at \( \underline{a} = (1/4, 1/4) \) and that

\[
\begin{align*}
    p_1 \frac{\partial f_1(\underline{a}/\ell_1)}{\partial a_1} &= 3/2, \\
    p_1 \frac{\partial f_1(\underline{a}/\ell_1)}{\partial a_2} &= 1/2, \\
    p_2 \frac{\partial f_2((1, 3/2) - \underline{a})\ell^{-1})}{\partial a_i} &= 1, \quad i = 1, 2.
\end{align*}
\]

By setting \( \underline{w}_j = (1, 1) \) we again obtain an equilibrium with common prices for each characteristic in all sectors. Firms in the first sector hire only on a subset of \([0, 1/2]\) say \([0, 1/4]\). Firms in the second sector hire in \( t \in (1/4, 1) \).
At an optimum where the constraint is binding and $0 < a_i < 1$, $i = 1, 2$, for some skill, say the first one, it follows from (4) that if

$$p_1 \frac{\partial f^1}{\partial a_1}(\bar{a}/\ell_1) > p_2 \frac{\partial f^2}{\partial a_1}((1, 1) - \bar{a})/\ell_2)$$

then

$$p_1 \frac{\partial f^1}{\partial a_2}(\bar{a}/\ell_1) < p_2 \frac{\partial f^2}{\partial a_2}((1, 1) - \bar{a})/\ell_2).$$
Now suppose sector one employs at least two types of workers with distinct ratios $x^1/x^2$ say $z$ and $\bar{z}$. If one such ratio is infinity (i.e., $z = \infty$) competition among firms in sector one requires $p_1 \frac{\partial f^1}{\partial a_1}(\bar{a}/\ell_1) = w_{11}$. But suppose both ratios are finite. Again, competition among firms in sector 1 requires that marginal profitability for all types of workers employed in the sector be zero so

\[
\left( p_1 \frac{\partial f^1}{\partial a_1}(\bar{a}/\ell_1) - w_{11} \right) z = w_{21} - p_1 \frac{\partial f^1}{\partial a_2}(\bar{a}/\ell_1)
\]

\[
\left( p_1 \frac{\partial f^1}{\partial a_1}(\bar{a}/\ell_1) - w_{11} \right) \bar{z} = w_{21} - p_1 \frac{\partial f^1}{\partial a_2}(\bar{a}/\ell_1)
\]

and since $z \neq \bar{z}$ it follows that $w_{11} = p_1 \frac{\partial f^1}{\partial a_1}$ and

$w_{21} = p_1 \frac{\partial f^1}{\partial a_2}$.
If firms in sector 2 employ at least two types of workers with distinct skill ratios, a parallel argument implies that

\[ w_{12} = p_2 \frac{\partial f^2}{\partial a_1} \]

and

\[ w_{22} = p_2 \frac{\partial f^2}{\partial a_2}. \]

By (5) and (6) it follows that \( w_{11} \neq w_{12} \) and \( w_{22} \neq w_{21} \) so that characteristic prices are not equal across sectors.
The Empirical Importance of Bundling
A Test of the Hypothesis of Equal Factor Prices Across All Sectors

- How to estimate the skill prices across sectors when there are unobserved skill prices?
- How to test equality of skill prices across sectors?
- Unobserved traits may be correlated with observed traits

\[ Y_{in} = \underbrace{x_{io} \psi_{no}}_{\text{observed}} + \left\{ \underbrace{x_{iu} \psi_{nu}}_{\text{unobserved}} + \varepsilon_{in} \right\}, \quad (7) \]

\[ i = 1, \ldots, I, \quad n = 1, \ldots, N. \]
• Allow for unobserved skills.
• Suppose that persons stay in one sector and we have $T$ time periods of panel data on those persons.
• Stack these into a vector of length $T$.
• Let $\kappa_u$ be the number of unobserved components.
• Let $\kappa_o$ be the number of observed components.
In matrix form we may write these equations for person $i$ as

$$Y_i = \tilde{x}_{io} \cdot \omega_o + \{\tilde{x}_{iu} \cdot \omega_u + \xi_i\}, \quad \text{for each sector } n \quad (8)$$

Following Madansky (1964), Chamberlain (1977) and Pudney (1982), assume $T \geq 2\kappa_u + 1$ and partition (8) into three subsystems:
(i) A basis subsystem of $\kappa_u$ equations from (8)

$$Y_{(1)} = \xi_i o \cdot \psi_o(1) + \{\xi_i u \cdot \psi_u(1) + \xi(1)\}, \quad n = 1, \ldots, N \quad (9a)$$

(ii) A second subsystem of equations all of which are distinct from the equations used in (i)

$$Y_{(2)} = \xi_i o \cdot \psi_o(2) + \{\xi_i u \cdot \psi_u(2) + \xi(2)\}, \quad n = 1, \ldots, N \quad (9b)$$

(iii) The rest of the equations (at least $\kappa_u$ in number)

$$Y_{(3)} = \xi_i o \cdot \psi_o(3) + \{\xi_i u \cdot \psi_u(3) + \xi(3)\}. \quad (9c)$$
Assuming that $\psi_{u(1)}$ is of full rank, the first system of equations may be solved for $\xi_{iu}$, i.e.,

$$
\xi_{iu} = \left[ Y_{(1)} - \xi_{io} \cdot w_o(1) - \varepsilon(1) \right] w_{u(1)}^{-1}.
$$

(10)
Substituting (10) into (9b), we reach

\[ Y_{(2)} = x_{io} \left[ w_{o(2)} - w_{o(1)} w_{u(1)}^{-1} w_{u(2)} \right] \]

\[ + Y_{(1)} w_{u(1)}^{-1} w_{u(2)} + \varepsilon_{(2)} - \varepsilon_{(1)} w_{u(1)}^{-1} w_{u(2)}. \]
• Letting $x_{i0(j)}$ be the observed characteristics for subsystem $j$, we reach our estimating equation

$$Y(2) = x_{io(2)}w_{o(2)} - x_{io(2)}w_{o(1)}w_{u(1)}^{-1}w_{u(2)}$$

$$+ Y(1)w_{u(1)}^{-1}w_{u(2)} + \varepsilon(2) - \varepsilon(1)w_{u(1)}^{-1}w_{u(2)}.$$  

• Use IV to instrument for $Y(1)$.  

• Find a lot of evidence against equality of factor prices across sectors.
<table>
<thead>
<tr>
<th>(1) Sector</th>
<th>(2) System MSE</th>
<th>(3) Test</th>
<th>(4) $F(DFN, DFD) =$</th>
<th>(5) Prob &gt; $F$</th>
<th>(6) Number of observations in each year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Durable vs. Nondurable</td>
<td>3.208210</td>
<td>1</td>
<td>(117, 1143) = 1.1448</td>
<td>0.1491</td>
<td>153</td>
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<td></td>
<td></td>
<td>2</td>
<td>(90, 1143) = 0.9213</td>
<td>0.6840</td>
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<td></td>
<td></td>
<td>3</td>
<td>(27, 1143) = 1.7777</td>
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<td>Manufacturing vs. Service</td>
<td>3.447400</td>
<td>1</td>
<td>(117, 3411) = 1.6754</td>
<td>0.0001</td>
<td>405</td>
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<td>2</td>
<td>(90, 3411) = 0.7336</td>
<td>0.9717</td>
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<td>3</td>
<td>(27, 3411) = 3.0062</td>
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<tr>
<td>Blue vs. White Collar</td>
<td>2.600956</td>
<td>1</td>
<td>(156, 6648) = 2.4197</td>
<td>0.0006</td>
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<td>(120, 6648) = 1.2943</td>
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<td>3</td>
<td>(36, 6648) = 3.0714</td>
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<td>North vs. South</td>
<td>2.299067</td>
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<td>(156, 7056) = 1.9586</td>
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<td>Manufacturing vs. Non-mfg</td>
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<td>3</td>
<td>(27, 5787) = 3.0978</td>
<td>0.0001</td>
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</tbody>
</table>

**Notes.**

1. Test 1 tests equality of the coefficients of (12) in both sectors.
2. Test 2 tests equality of the coefficients associated with observed characteristics in (12).
3. Test 3 tests equality of the coefficients associated with the unobserved characteristics in (12) ($w_{u(1)}$, $w_{u(2)}$).
Notes.

1. Test 1 tests equality of the coefficients of (12) in both sectors.
   Test 2 tests equality of the coefficients associated with observed characteristics in (12).
   Test 3 tests equality of the coefficients associated with the unobserved characteristics in (12) \((w_u^{-1}, w_u)\).

2. Durable: Metal Industries, Machinery including Electrical, Motor Vehicles and other Transportation Equipment, other durables.

   Non Durable: Food, Tobacco, Textile, Paper, Chemical and other Non Durables.

   Manufacturing: All Durable and Non Durable plus “manufacturing unknown”.

   Services: Retail Trade, Wholesale Trade, Finance, Insurance, Real Estate, Repair Service, Business Service, Personal Service, Amusement, Recreation and Related Services, Printing, Publishing and Allied Services, Medical and Dental Services, Educational Services, Professional and Related Services.


   White Collar: Professional, Technical and Kindred; Managers, Officials and Proprietors; Self Employed Businessmen; Clerical and Sales Work.

   Blue Collar: Craftsmen, Foremen and Kindred Workers; Operatives and Kindred Workers; Labourers and Service Workers, Farm Labourers.
<table>
<thead>
<tr>
<th>Sector</th>
<th>(2) System MSE</th>
<th>(3) Test</th>
<th>(4) $F(DFN, DFD)$</th>
<th>(5) Prob &gt; $F$</th>
<th>(6) Number of observations in each year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Durable vs. Nondurable</td>
<td>1.480446</td>
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<td>(144, 1089) = 1.2902</td>
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<td>2</td>
<td>(108, 1089) = 1.1722</td>
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<td>3</td>
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<td>(36, 3357) = 6.6334</td>
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<td>Blue vs. White Collar</td>
<td>3.830300</td>
<td>1</td>
<td>(192, 6576) = 1.7228</td>
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<td>2</td>
<td>(144, 6576) = 1.3400</td>
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<td>3</td>
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<td>0.0003</td>
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<td>North vs. South</td>
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<td>(192, 6984) = 1.9893</td>
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<td>(48, 1836) = 2.3018</td>
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<td>Manufacturing vs. Non-mfg.</td>
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<td>(132, 1836) = 1.4107</td>
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<td>(48, 1836) = 2.0701</td>
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<tr>
<td>Sector</td>
<td>MSE</td>
<td>Test</td>
<td>$F(DFN, DFD)$ =</td>
<td>Prob $&gt; F$</td>
<td>Observations</td>
</tr>
<tr>
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<tr>
<td>Blue vs. White Collar</td>
<td>1.573852</td>
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<td>(228, 6912) = 2.0534</td>
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<td>(168, 6912) = 1.6639</td>
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<td>North vs. South</td>
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<td>2</td>
<td>(168, 6504) = 2.2027</td>
<td>0.0001</td>
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<td>3</td>
<td>(60, 6504) = 10.0017</td>
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</table>
APPENDIX

For the 3 factor models we adopt the following basis:

<table>
<thead>
<tr>
<th>Years for wages ($Y_{(2)}$)</th>
<th>Basis years</th>
</tr>
</thead>
</table>

For the 4 factor models we adopt the following choice of basis:

<table>
<thead>
<tr>
<th>Years for wages ($Y_{(2)}$)</th>
<th>Basis years</th>
</tr>
</thead>
</table>

For the 5 factor models we adopt the following choice of basis:

<table>
<thead>
<tr>
<th>Years for wages ($Y_{(2)}$)</th>
<th>Basis years</th>
</tr>
</thead>
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