Micro Data and General Equilibrium Models
(Extract on Frisch Demand)

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Econ 350
This draft, March 10, 2013 2:52pm
3. Micro evidence

3.1. Introduction

This part of the chapter presents additional evidence from the microeconomic literature on the parameter values that are required to implement the dynamic general equilibrium (DGE) models analyzed in Sections 1 and 2. Several conceptually distinct labor supply and consumption demand elasticities are presented. We discuss the problem of research synthesis and issue a warning against uncritical use of the existing micro evidence in standard general equilibrium models. We also present further evidence on preference heterogeneity. We start by defining a variety of conceptually different elasticities that are frequently confused in both the micro and macro literatures.
3.2. Defining elasticities

When considering the estimation of parameters for ultimate use in a DGE model, it is important to keep track of exactly what is being held constant (the “conditioning variables”) in the process of estimation in order to ascertain whether the parameter being estimated corresponds to the parameter required in a general equilibrium model. We illustrate this point by considering a simple model of consumption and labor supply, but our discussion applies much more generally. We derive three frequently estimated elasticities typically formulated for a model in which an agent sells labor on a spot market without any transactions costs or fixed costs of employment. Although these elasticities are often referred to by the same name, they correspond to different choices of conditioning variables and hence distinct conceptual experiments. We then consider aggregate labor supply response measures that account for heterogeneity and dichotomous work–no work decisions. In later sections we present some parameter estimates of these elasticities taking care to specify which elasticity is being estimated.
Suppose that in a given period $t$ a person chooses current non-durable consumption $c_t$ and hours of market work $h_t = T - l_t$. Preferences are intertemporally additive but the within-period utility function $U(c_t, h_t)$ is not. In what follows we separate the within-time-period decision of how to allocate consumption and leisure given current period net expenditures from the intertemporal savings decision. This leads us to define “total net expenditure” or “net dissavings” $e_t = p_t c_t - w_t h_t$, where $p_t$ is the time $t$ price of consumption and $w_t$ is the time $t$ wage rate. These prices can be denominated in any convenient unit of account, including time $t$ dollars. Deciding how much to consume and how much to work for a given amount of net dissavings is one part of the decision problem confronting the consumer.
In a model with a more fully specified security market, the link between net dissavings and the market opportunities for investment is clearer. As we discussed in Section 1, it is conventional in the RBC literature to consider a wide array of alternative security markets. Following Heckman (1974) the impact of the allocation of \( e_t \) over time is conveniently determined by the life-cycle evolution of the shadow price \( \lambda_t \) of the net dissavings: the marginal utility of income. Changing the specification of the security market environment alters the evolution of \( \{\lambda_t\} \). For instance, suppose that the consumer/investor has access to a one-period risk-free security with rate of return \( r_{t+1} \). Then the marginal utility of income satisfies the stochastic difference equation:

\[
\lambda_t = \beta (1 + r_{t+1}) E(\lambda_{t+1} \mid I_t),
\]  

(3.1)

where \( \beta \) is the subjective discount factor, provided there is no binding borrowing constraint [MaCurdy (1983)]. Under the permanent income restriction that \( \beta (1 + r_{t+1}) = 1 \), this results in the familiar conclusion that the marginal utility of income is a martingale [MaCurdy (1978), Hall (1978)]. Including risky securities at the same time imposes further restrictions on the evolution of \( \{\lambda_t\} \). When the rate of return is a risky asset, Equation (3.1) becomes

\[
\lambda_t = \beta E \left[ (1 + r_{t+1})\lambda_{t+1} \mid I_t \right].
\]
Including additional risky securities we obtain

\[ \lambda_t = \beta E[(1 + r_{t+1}^j)\lambda_{t+1} \mid I_t] \]

for \( j = 1, \ldots, J \) where \( J \) is the number of securities. The presence of short sale constraints may convert these arbitrage equalities into strict inequalities. In the limiting complete market case, the ratio \( \beta(\lambda_{t+1}/\lambda_t) \) is the same for all consumers and is equal to the market stochastic discount factor for pricing single-period securities described in Equation (1.7) in Section 1 [Altug and Miller (1990, 1998)]. We next consider Frisch demand functions that condition on the shadow price \( \lambda_t \). While a multiplier \( \lambda_t \) can be defined for all environments, its interpretation is more complicated in environments with borrowing or short sale constraints.
3.2.1. Frisch demands

Assuming interior solutions for consumption and hours \((c_t > 0 \text{ and } T > h_t > 0)\), and Equation (3.1), or some generalization consistent with no corner solutions in
intertemporal financial transfers, the household optimal consumption and hours of work satisfy two first-order conditions:

\[ U_c(c_t, h_t) = \lambda_t p_t, \]  
\[ U_h(c_t, h_t) = -\lambda_t w_t. \]  

(3.2)

(3.3)

If the utility function \( U(c_t, T - l_t) \) is strictly concave in consumption and leisure, we can invert Equations (3.2) and (3.3) to give the Frisch (or \( \lambda \)-constant) consumption and labor supply functions (where now we drop the \( t \) subscripts):

\[ c = c(p, w, \lambda), \]  
\[ h = h(p, w, \lambda). \]  

(3.4)

(3.5)
From the integrability conditions, these functions are homogeneous of degree zero in the price, the wage and the inverse of \( \lambda \); symmetric \((c_w = -h_p)\); and satisfy negativity (which implies \( c_p < 0 \) and \( h_w > 0 \)).

A common assumption in both the empirical literature in microeconomics and in the macroeconomic literature is that consumption and labor supply are additively separable within the period; this is equivalent to assuming that \( c_w = h_p = 0 \). We do not invoke this assumption here and we note below that the micro evidence speaks against it.

The case in which an individual or household chooses not to work: \( h = 0 \), is also of considerable interest. In the absence of fixed costs for entry and exit, a person chooses not to work if:

\[
U_c(c, 0) = \lambda p, \quad U_h(c, 0) < -\lambda w,
\]

that is, if the reservation value of leisure at zero hours of work is greater than the market wage. We say more about this case in Section 3.2.4.
Associated with the Frisch consumption and labor supply functions are the Frisch (or \( \lambda \)-constant) price and wage elasticities:

\[
\varphi(p, w, \lambda) = \frac{\partial \ln c}{\partial \ln p} = c_p(p, w, \lambda) \frac{p}{c},
\]

\[
\theta(p, w, \lambda) = \frac{\partial \ln h}{\partial \ln w} = h_w(p, w, \lambda) \frac{w}{h}. 
\]

(3.6)

These elasticities consider changes in demands and supplies for a particular good when its own price is changed but other prices are held constant. For instance, \( \theta \), the Frisch (or \( \lambda \)-constant) elasticity of labor with respect to the nominal wage, holds the nominal price of consumption constant and hence also measures hours response to
increases in the real wage. As we will see, the conditioning on $\lambda$ gives both elasticities an intertemporal character.

To construct the *intertemporal elasticity for consumption* we consider

$$
\eta(p, w, \lambda) = \frac{\partial \ln c}{\partial \ln \lambda} = c_\lambda(p, w, \lambda) \frac{\lambda}{c}.
$$

$\lambda_t$ is the marginal utility of income in period $t$. 
There is an obvious counterpart for labor supply. Assuming no binding borrowing or short sale constraints, the change in $\lambda$ can be thought of as arising from a change in the interest rate or any other factor that alters $\lambda$ through forward-looking relation (3.1). In contrast to $\eta$, $\varphi$ is defined for a change that holds $\lambda$ fixed. For this reason, $\varphi$ sometimes is referred to as an intertemporal elasticity of substitution for consumption When the utility function $U$ is additively separable between consumption and hours, then $\varphi$ and $\eta$ coincide, but in general they do not. Instead, by the homogeneity of degree zero of the Frisch demand functions, we have the relation

$$c_p (p, w, \lambda) \frac{p}{c} + c_w (p, w, \lambda) \frac{w}{c} = c_\lambda (p, w, \lambda) \frac{\lambda}{c} = \eta(p, w, \lambda),$$

or

$$\varphi (p, w, \lambda) + c_w (p, w, \lambda) \frac{w}{c} = \eta(p, w, \lambda).$$
Thus, the intertemporal elasticity $\eta$ can also be viewed as the consumption response when both the price and wage change proportionately.
3.2.2. Other demand functions

Within period elasticities can be defined even if Euler inequalities rather than Euler equations characterize consumer behavior. Assuming equalities hold, the within-period elasticities can be derived by substituting total net expenditures $e = pc - wh$ for $\lambda$, by inverting the following expression:

\[ e = pc(p, w, \lambda) - wh(p, w, \lambda) = \psi(p, w, \lambda) \Rightarrow \lambda = \mu(p, w, e). \]
Substituting in the Frisch consumption and labor supply functions we obtain the within-period *uncompensated* or *Marshallian* consumption and labor supply functions:

\[ c = c [p, w, \mu (p, w, e)] = c^* (p, w, e), \]  

\[ h = h [p, w, \mu (p, w, e)] = h^* (p, w, e). \]  

(3.8)  

(3.9)

This derivation is more restrictive than necessary because it assumes that Frisch demands exist. An alternative derivation that stresses the more general nature of these demand functions maximizes within-period utility subject to a period-specific budget constraint that may be augmented beyond (or below) current period earnings. Under this interpretation, \( e \) is the supplement to current earnings that governs within-period choices and \( e \) may be either exogenously or endogenously determined.
Under either interpretation, the parameters of $c^*(\cdot)$ and $h^*(\cdot)$ can be estimated using only cross-section data. However, they do not recover all the parameters of the original functions $c(\cdot)$ and $h(\cdot)$ except under very stringent assumptions. In particular, because they condition on the within-period allocation of expenditures, they are uninformative on interperiod substitution unless strong functional form assumptions are maintained. On the other hand, those parameters that are identified from Equations (3.8) and (3.9) are robust to misspecifying the asset market structure used to characterize the evolution of the marginal utility of income. For example, these relationships are valid whether or not the consumer is up against a borrowing constraint.
A third labor supply function can be derived directly by equating the intratemporal marginal rate of substitution between hours and consumption to the real wage. By forming ratios of Equations (3.2) and (3.3), we eliminate the marginal utility of income \( \lambda \). Solving for labor supply in terms of the price of consumption, the wage and consumption gives:

\[
h = h^{**}(p, w, c),
\]

(3.10)

where now we condition labor supply responses on consumption. Alternatively, we could solve for consumption by conditioning on the wage and hours. Since these demand functions are readily derived from the marginal rate of substitution first-order condition, they are known as m-demand/supply functions [see Browning (1998)]. They are valid whether or not the intertemporal Euler equations are binding. As noted in Section 1, these demand relations are also among those used by macroeconomists to calibrate parameters from steady-state relations using time series averages.
The two conditional labor supply wage elasticities associated with Equations (3.10) and (3.9) generally differ from each other and from the corresponding Frisch elasticity given in Equation (3.6). Assuming that there are no binding constraints connecting transfers among periods, the three demand functions are connected by the identities

\[ h(p, w, \lambda) \equiv h^*[p, w, \psi(p, w, \lambda)] \equiv h^{**}[p, w, c(p, w, \lambda)] \]  

(3.11)

so that the wage effects are related by differentiating with respect to \( w \):

\[ h_w(p, w, \lambda) = h^*_w(p, w, e) + h^*_e(p, w, e)\psi_w(p, w, \lambda) \]

\[ = h^{**}_w(p, w, c) + h^{**}_c(p, w, c)c_w(p, w, \lambda). \]  

(3.12)
The first equation states that the Frisch (or $\lambda$-constant) wage responses for labor supply are equal to the within-period wage response holding resource flow constant plus an intertemporal net savings response that accounts for how savings is altered by the wage change. The second equation shows that the Frisch wage effect can be decomposed into a within-period effect of wages on hours holding current consumption constant plus an intertemporal response of consumption to wages.

From cross-section data on consumption expenditures and labor supply we can estimate $h^*(p, w, e)$ and $h^{**}(p, w, c)$ assuming either exogeneity of the conditioning variables or access to valid instruments for the endogenous regressors. Each demand equation can be used to bound the intertemporal Frisch response $h_w(p, w, \lambda)$. 
More precisely, from Equation (3.7) we have

$$\psi_w(p, w, \lambda) = pc_w(p, w, \lambda) - wh_w(p, w, \lambda) - h.$$ 

The sign of $\psi_w(\cdot)$, the effect of wages on borrowing, or dissaving, is ambiguous, if as is widely assumed, consumption and labor supply are Frisch complements (so $c_w(\cdot) > 0$). However, it is unlikely that the cross effect on the right hand side will outweigh the other two terms. In a period of high wages it seems plausible that savings increase, rather than decrease or remain constant. Thus $\psi_w < 0$. In this case, $h_w(\cdot) \geq h^*_w(\cdot)$ since $h^*_c(\cdot)$ is negative if leisure is a normal good. The m-supply response $h^{**}_w(p, w, c)$ gives an upper bound for the Frisch response $h_w(\cdot)$ if leisure and consumption are normal (so that $h^{**}_c(\cdot) < 0$) and consumption and labor supply are complements (so $c_w(\cdot) > 0$). Thus $h^{**}_w < h_w < h^*_w$ so we can bound $h_w$ from the cross-sectional relationships. Obviously if consumption and leisure are additively separable within periods, the m-supply response function is the Frisch response. In general, we must take care to specify exactly what is being held constant (the marginal utility of income, total net expenditure or consumption) in the empirical study we examine when we use an estimated wage elasticity The empirical literature presents estimates of all of these elasticities and more, as we discuss below, and often does not distinguish among them.
Ordering on Frisch and Compensated (Within Period) Elasticities

\[ h_w \geq 0 \quad \text{From normality and concavity} \]

\[ h_w = \underbrace{s^*}_{\text{Marshallian uncompensated effect}} + h \frac{\partial h^*}{\partial e} \]

\[ + \frac{\partial h^*}{\partial e} \left( \psi_w - h - wh_w + pc_w \right) \]

Marginal effect of change in \( w \) on net borrowing in the period

\[ (\text{Slutsky compensated within-period effect}) \]

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\[ c_w > 0 \] (consumption and leisure substitutes \( \iff \) consumption and hours of work complements)

\[ c_w = 0 \] (additive separability)

\[ c_w < 0 \] (consumption and leisure complements)

If \( c_w \leq 0 \),

\[ h_w \geq s^* \geq h^*_w \]

\begin{align*}
\text{Frisch} & \quad \text{Slutsky compensated} & \quad \text{Marshallian}
\end{align*}
More generally (noting terms that cancel)

\[
    h_w = \left(\begin{array}{c}
        (+) \\
        (-)
    \end{array}\right) s^* + \left(\begin{array}{c}
        \partial h^* \\
        \partial e
    \end{array}\right) ( -wh_w + pc_w )
\]

effect on borrowing of wage increase

clearly \( wh_w \geq 0 \) but what about \( pc_w \)?

\( pc_w \) is the effect on borrowing arising from wage increases in consumption (holding \( \lambda \) constant).
\( wh_w \) is the income-generating effect on borrowing of the wage increase.
\( pc_w \) is the consumption-generating effect of the wage increase.
3.2.3. An example

To make the discussion of Section 3.2.2 more concrete, consider the following utility function which, as we have previously noted, is sometimes used in DGE models:

\[
U(c, h) = \frac{c^\sigma (T - h)^{(1-\sigma)}}{(1-\rho)} - 1; \quad 0 < \sigma < 1, \quad \rho > 0, \tag{3.13}
\]

where \(T\) is the time available for work. The conditions on the admissible parameter values are produced by monotonicity and strict concavity. The associated Frisch consumption and labor supply functions are:

\[
\ln c = \alpha_c + \beta_c \ln p + \gamma_c \ln w + (\beta_c + \gamma_c) \ln \lambda, \tag{3.14}
\]
\[ \ln l = \ln (T - h) = \alpha_h + \beta_h \ln p + \gamma_h \ln w + (\beta_h + \gamma_h) \ln \lambda, \]  

(3.15)

where

\[ \beta_c = \frac{(\sigma \rho - \sigma - \rho)}{\rho}, \quad \gamma_c = -\frac{(1 - \sigma)(1 - \rho)}{\rho}, \]

\[ \beta_h = -\frac{\sigma(1 - \rho)}{\rho}, \quad \gamma_h = \frac{\sigma(1 - \rho) - 1}{\rho}. \]
The monotonicity and concavity conditions on the utility function imply that $\beta_c$ and $\gamma_h$ are both negative. Note as well that

$$(\beta_c + \gamma_c) = (\beta_h + \gamma_h) = -\frac{1}{\rho} < 0,$$

so that both consumption and leisure are normal goods (that is, increases in lifetime wealth lead to decreases in the marginal utility of income and consequent increases in both consumption and leisure). When $\rho = 1$, the utility function $U$ is additively separable in consumption and leisure:

$$U(c, h) = \sigma \ln(c) + (1 - \sigma) \ln(T - h),$$

and as a consequence, $\gamma_c = \beta_h = 0$ and $\gamma_h = \beta_c = -1$.

The Frisch labor supply wage elasticity is given by:

$$\frac{\partial \ln h}{\partial \ln w} = -\gamma_h \frac{(T - h)}{h}. \quad (3.16)$$
The Frisch elasticity for consumption holding the wage constant and the marginal utility of income constant, is given by $\beta_c$; and the elasticity of intertemporal substitution is given by $-1/\rho$, which is the coefficient on $\ln \lambda$ in Equation (3.14). This latter elasticity can alternatively be viewed as the *elasticity of intertemporal substitution holding the real wage constant*. This may be seen by rewriting the consumption function as

$$\ln c = \alpha_c + (\beta_c + \gamma_c) \ln p + \gamma_c \ln \frac{w}{p} + (\beta_c + \gamma_c) \ln \lambda. \tag{3.17}$$

Both $\ln p$ and $\ln \lambda$ now have a common coefficient:

$$\beta_c + \gamma_c = -\frac{1}{\rho},$$

which shows that the two elasticities are the same.
The consumption elasticities $\beta_c$ and $\beta_c + \gamma_c$ result from two different conceptual experiments, as reflected by their different conditioning variables. They coincide only in very special cases. When the consumer does not care about non-work time (leisure) ($\sigma = 1$) only the consumption equation (3.14) is relevant and $\gamma_c = 0$. In this case both consumption elasticities coincide and are given by $-1/\rho$. This is the usual definition for an iso-elastic consumption utility function. Alternatively, when $U(c, h)$ is additively separable ($\rho = 1$), $\gamma_c$ is again zero and $\beta_c = -1$. When consumption and labor supply are not additively separable within the period we have two distinct Frisch intertemporal substitution elasticities for consumption one holding the current wage constant ($\beta_c$) and one holding the real wage constant ($\beta_c + \gamma_c$).
The utility function given in Equation (3.13) is very restrictive since it confounds intertemporal substitution conditions – a high value for $\rho$ implies a high propensity to substitute across time – and within-period substitution possibilities – the cross elasticities ($\beta_h$ and $\gamma_c$) are positive if and only if $\rho > 1$. When $\rho > 1$, market work and consumption are (Frisch) complements; that is, a rise in the wage leads to an increase in both consumption and labor supply; market goods substitute for the home production foregone when someone works. As discussed in the previous subsection, we can also derive within-period labor supply functions that condition on either net dissaving or consumption For the utility function (3.13), the $e$-constant demand function is not very illuminating but the $c$-constant function is given by

$$
\ln (T - h) = \ln \left( \frac{\sigma}{1 - \sigma} \right) + \ln (c) - \ln \left( \frac{w}{p} \right).
$$

(3.18)
This equation clearly demonstrates that we cannot generally identify the preference parameters for the intertemporal allocation from within-period information. Indeed, in this case the parameter $\rho$ cannot be identified from the $c$-constant function and thus none of the parameters of the Frisch consumption and labor supply functions can be deduced. While parameter $\sigma$ can be inferred from this intratemporal relation, this particular choice of functional form implicitly imposes the requirement that at a fixed real wage, consumption and leisure move together – a prediction that is at odds with the evidence surveyed in Section 3.3.\textsuperscript{41}
3.2.4. *The life-cycle participation decision*

The discussion so far assumes that solutions for consumption and leisure are interior. Such an assumption is congenial to the representative agent approach to macroeconomics but is grossly inconsistent with the microeconomic evidence on labor supply summarized by Pencavel (1986) and Killingsworth and Heckman (1986). Summarizing the Ph.D. research of Coleman (1984), Heckman (1984) noted that even for prime age males, variations in employment contribute about 50% of the total variation in person hours over the business cycle. A central finding of the modern labor supply literature summarized in Heckman (1978, 1993) and Blundell and MaCurdy (1999) is that most of the curvature in the labor supply–wage relationship comes from choices at the extensive (entry–exit) margin.
Accounting for entry and exit decisions forces analysts to introduce heterogeneity among agents. It is implausible that all agents either work in a period or do not work in a given period. Some mechanism must be introduced to account for why some agents work while others do not. In the macro literature building on Rogerson (1988), an assumption of fixed costs of work or some source of non-convexity is introduced so that it is optimal for individual agents to work either full time or not at all. The mechanism used to allocate work across people is a lottery embedded in a complete contingent claims market. [See Rogerson (1988) or the survey in Hansen and Prescott (1995).] Ex ante identical persons get different draws from the lottery. Draws are independent over time. Under additive separability in consumption and leisure, the winners of the lottery are those denied work; they get the same consumption bundle as workers but enjoy more leisure than workers. Rogerson (1988) shows how lotteries can be priced in a complete market RBC model of the sort considered in Section 142. Under conditions presented in his paper, a decentralized mechanism exists and a competitive equilibrium can be supported by the pricing system.
This description of the employment allocation mechanism strains credibility and is at odds with the micro evidence on individual employment histories. Heckman and Willis (1977), Heckman (1982) and Clark and Summers (1979) document that employment indicator variables for persons are highly correlated over time – contrary to the Bernoulli assumption implicit in Rogerson (1988) and Hansen and Prescott (1995). Over long stretches of time, some people work all of the time while others never work. Heckman (1982) explicitly tests and rejects the Bernoulli assumption. This persistence in employment status remains even after controlling for commonly observed characteristics such as education, age and work experience. Persistence in the employment or non-employment state is a central feature of the micro data. At a minimum, individual-specific lotteries with outcomes strongly correlated over time are required to account for the micro data on employment – histories complicating the analysis of equilibrium lottery pricing. The heterogeneity in employment experience among persons of the same apparent demographic and productivity characteristics suggests that problems with adverse selection and moral hazard are likely to render the competitive lotteries of the sort analyzed by Rogerson (1988) infeasible.
The micro literature explains cross-section heterogeneity in employment experiences, and the persistence of employment status over time, by introducing temporally invariant person-specific unobserved heterogeneity. The evidence in Heckman and Willis (1977) and Heckman (1982) indicates that such invariant components account for much of the persistence in employment status over time. Invariant unobservables can be introduced into all three types of demand functions analyzed in Section 3.2.2.

Within the Frisch framework, if we abstract from any fixed costs of work or other non-convexities, a person will not work in period $t$ if

$$U_c(c_t, 0) = \lambda_t p_t$$

(3.19)

and

$$U_h(c_t, 0) < -\lambda_t w_t.$$  

(3.20)
We can solve out for the virtual wage, $w^*_t$, that makes inequality (3.20) an exact equality at zero hours of work and use $w^*_t$ in the Frisch consumption demands. Thus, unless there is contemporaneous additive separability, estimation of the consumption demand equation will depend on virtual wages which equal actual wages if a person works. Similarly, in the absence of contemporaneous additive separability, the employment decision will depend on $p_t$ as well as on $\lambda_t$ and $w_t$. To see this, solve the first-order condition for consumption (3.19) for $c_t(\lambda_t, p_t; h_t = 0)$ and substitute into inequality (3.20). This produces an equation characterizing life-cycle employment that depends on $p_t$, among other factors.
The employment decision is discontinuous in terms of $w_t$; below or at $w_t^*$, persons will not work; above $w_t^*$, persons work at age $t$. Introduce a person-specific time-invariant random variable $\varepsilon$ to account for heterogeneity in the population. This is unobserved by the econometrician. Instead the econometrician observes a vector $X$ of demographic characteristics that partially predict $\varepsilon$. Associated with this random variable is a distribution of types in the population. Let $d_t = 1$ if a person is employed at time $t$; $d_t = 0$ otherwise. Accounting for heterogeneity, $\varepsilon$, which for simplicity is assumed to be scalar, we may write this inequality as

$$d_t = 0 \text{ if } U_h(c_t(\lambda_t(\varepsilon), p_t; h_t = 0), 0; \varepsilon) \leq -\lambda_t(\varepsilon)w_t,$$

$$d_t = 1 \text{ otherwise},$$

or more succinctly

$$d_t = 1[ U_h(c(\lambda_t(\varepsilon), p_t; h_t = 0), \varepsilon) > -\lambda_t(\varepsilon)w_t ],$$

where $1(\cdot)$ is the indicator function and where we note that $\lambda_t(\varepsilon)$ does not depend on $w_t$ if the inequality is strict\(^{43}\). However, standard results in consumer theory demonstrate

\(^{43}\) The assumption that $\varepsilon$ is scalar is only a simplifying device. It is not strictly required. Note that $\lambda_t(\varepsilon)$ is a function of $\varepsilon$ and initial assets as well as prices and wages in periods where persons work. For simplicity of notation we only exhibit the dependence on $\varepsilon$. 
that $\lambda_t(\varepsilon)$ depends on $\varepsilon$ and current and future prices and wages for periods in which persons work and consume. The set of $\varepsilon$ values for which $d_t = 0$ for wage $w_t$ is thus given by

$$e_t = \{ \varepsilon \mid U_h[c_t(\lambda_t(\varepsilon), p_t; h_t = 0, 0; \varepsilon) \leq -\lambda_t(\varepsilon)w_t] \},$$

which depends implicitly on all prices and wages over the life cycle in periods outside of $t$ in which the consumer works and consumes as well as initial endowments. Let the boundary of the set $e_t$ for persons with potential market wage $w_t$ be

$$B(e_t) = \{ \varepsilon \mid U_h[c_t(\lambda_t(\varepsilon), p_t; h_t = 0, 0; \varepsilon) = -\lambda_t(\varepsilon)w_t] \}.$$
Assume, for simplicity, that prices and wages are independent of ε and demographic characteristics X do not enter preferences directly, except through their effect on ε. These assumptions simplify the notation and are easily relaxed. Then, in period t, the proportion not working is

$$\Pr(d_t = 0 \mid X) = \int_{\varepsilon_i} dF(\varepsilon \mid X),$$

where $F(\varepsilon \mid X)$ is the conditional distribution of ε. The life-cycle wage response of participation is

$$\frac{\partial \Pr(d_t = 0 \mid X)}{\partial w_t} = f(B(\varepsilon_i) \mid X) \frac{\partial B(\varepsilon_i)}{\partial w_t},$$

(3.21)

where we are assuming that there is only one boundary point, an upper boundary point, and that the distribution is continuous at that point with density $f'$:

$$\left. \frac{dF(\varepsilon \mid X)}{d\varepsilon} \right|_{\varepsilon = B(\varepsilon_i)} = f(B(\varepsilon_i) \mid X).$$
For more general boundary sets, we require a notion of the density added or subtracted as the boundary is changed by the wage\textsuperscript{44}. Aggregating over age, wage and $X$ groups, as in Section 2, produces the aggregate proportion of people who do not work.

\textsuperscript{44} We require evaluation of the limit

$$
\lim_{\Delta w_i \to 0} \frac{\int_{\varepsilon_i(w_i + \Delta w_i)} dF(\theta \mid X) - \int_{\varepsilon_i(w_i)} dF(\theta \mid X)}{\Delta w_i},
$$

where we make the dependence of $\varepsilon_i$ on $w_i$ explicit. This expression is easily generalized to allow for vector $\varepsilon$. 
Aggregate labor supply is constructed accounting for both the choice at the intensive margin and choice at the extensive margin. See Heckman (1978) or Pencavel (1986) for more details. Aggregate employment–wage parameters combine preferences and distribution parameters and cannot be directly compared with the interior solution elasticities unless the distribution of taste parameters is accounted for\textsuperscript{45}.

A similar analysis can be performed for the $e$-constant and $c$-constant demand functions\textsuperscript{46}. In each case we can define a reservation wage and can define a set of $\varepsilon$ values such that for persons with a given $e_t(\varepsilon)$ or $c_t(\varepsilon)$, respectively, and for the other prices and $X$ variables, persons work or do not work in period $t$. In general $e_t$ and $c_t$ depend on $\varepsilon$, just as $\lambda_t$ depends on $\varepsilon$. Employment and non-employment proportions are determined by integration of $dF(\varepsilon \mid X)$ over the appropriate sets. We note, however, that all three interpretations of the leisure demand function produce the same aggregate employment elasticity provided that all three exist\textsuperscript{47}. We now turn to the empirical evidence on these elasticities starting with the $eis$ for consumption.