The Koyck (1954) - Chetty (2012)
Short Run vs. Long Run Response

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\[ y^* = \text{optimal level (Long Run)} \]
\[ y_t = \text{actual level} \]
\[ y_{t-1} = \text{previous level (already realized)} \]

Pick \( y_t \) to minimize the following loss function
\[ \frac{1}{2} \phi (y^* - y_t)^2 + \frac{1}{2} (y_t - y_{t+1})^2 \eta \]

\text{cost of being away from optimum} \quad \text{cost of adjustment}^2 \quad \phi, \eta > 0

\text{FOC: } - \phi (y^* - y_t) + \eta (y_t - y_{t-1}) = 0

\text{FOC: } y_t = \frac{\phi}{\phi + \eta} y^* + \frac{\eta y_{t-1}}{\phi + \eta}
Let $y^* = \tau(x)$

$$\frac{\partial y_t}{\partial x} = \frac{\phi}{\phi + \eta} \frac{\partial y^*}{\partial x} = \frac{\phi}{\phi + \eta} \frac{\partial \tau(x)}{\partial x}$$

$< 1$