Marschak’s Maxim

James J. Heckman

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Counterfactuals, Causality and Structural Econometric Models

• The literature on policy evaluation in economics sometimes compares “structural” approaches with “treatment effect” or “causal” models.

• These terms are used loosely.

• This section formally defines “structural” models and uses them as devices for generating counterfactuals.
Counterfactuals, Causality and Structural Econometric Models: Modeling the Choice of Treatment

- Parallel to causal models for outcomes are causal models for the choice of treatment.

- Consider *ex ante* personal valuations of outcomes based on expectations of gains from receiving treatment $s$:

\[
E(R(Y(s, \omega), C(s, \omega), Q(s, \omega), \omega) | I_\omega), s \in S,
\]

where as before $C(s, \omega)$ is the price or cost agent $\omega$ must pay for participation in treatment $s$. 
• We decompose \( C(s, \omega) \) into observables and unobservables.

• We write

\[
C(s, \omega) = K(W(s, \omega), \eta(s, \omega)).
\]

• We allow utility \( R \) to be defined over the characteristics that generate the treatment outcome (e.g., quality of teachers in a schooling choice model) as well as attributes of the agent.

• In parallel with the \( g_s \) function generating the \( Y(s, \omega) \), we write

\[
R(Y(s, \omega), C(s, \omega), Q(s, \omega), \omega)
= f(Y(s, \omega), W(s, \omega), Q(s, \omega), \eta(s, \omega), \omega).
\]
• We may keep $Q(s, \omega)$ implicit and use $f_s$ functions instead of $f$.

• In the Roy model, $R = Y_1 - Y_0 - C$ is the agent’s subjective evaluation of treatment.

• The agent computes expectations against his/her subjective distribution of information.

• The $\mathcal{I}_\omega$ are the causal factors for agent $\omega$. 
• In a utility maximizing framework, choice $\hat{s}$ is made if $\hat{s}$ is maximal in the set of valuations of potential outcomes:

$$\{E(R(Y(s, \omega), C(s, \omega), Q(s, \omega), \omega) \mid \mathcal{I}_\omega), s \in S\}.$$ 

• In this interpretation, the information set plays a key role in specifying agent preferences.
Counterfactuals, Causality and Structural Econometric Models: The Econometric Model vs. the Neyman–Rubin Model

- Many statisticians and social scientists invoke a model of counterfactuals and causality attributed to Donald Rubin by Paul Holland (1986), but which is actually due to Neyman (1923).

- Neyman and Rubin postulate counterfactuals \{Y(s, \omega)\}_{s \in S} without modeling the factors determining the \( Y(s, \omega) \) as we have done using the econometric or “structural” approach.

- Rubin and Neyman offer no model of the choice of which outcome is selected.
Counterfactuals, Causality and Structural Econometric Models: Nonrecursive (Simultaneous) Models of Causality

- A system of linear simultaneous equations captures interdependence among outcomes $Y$.

- For simplicity, in this discussion we focus on *ex post* outcomes, and so we ignore revelation of information over time.
Counterfactuals, Causality and Structural Econometric Models: Nonrecursive (Simultaneous) Models of Causality

- A system of linear simultaneous equations captures interdependence among outcomes $Y$.

- For simplicity, in this discussion we focus on *ex post* outcomes, and so we ignore revelation of information over time.
• Write the standard model of simultaneous equations in terms of parameters \((\Gamma, B)\), observables \((Y, X)\) and unobservables \(U\) as

\[
\Gamma Y + BX = U, \quad E(U) = 0, \quad (1)
\]

where \(Y\) is a vector of endogenous and interdependent variables, \(X\) is exogenous \((E(U | X) = 0)\), and \(\Gamma\) is a full rank matrix.

• Equation systems like (1) are sometimes called “structural equations.”
• The $Y$ are “internal” variables determined by the model and the $X$ are “external” variables specified outside the model.

• Assume the model is complete ($\Gamma^{-1}$ exists), gives unique $Y$.

• Reduced form is $Y = \Pi X + R$ where $\Pi = -\Gamma^{-1} B$ and $R = \Gamma^{-1} U$. 

The “structure” is \((\Gamma, B), \Sigma_U\), where \(\Sigma_U\) is the variance-covariance matrix of \(U\).

Assume that \(\Gamma, B, \Sigma_U\) are invariant to general changes in \(X\) and translations of \(U\).

Without restrictions, ceteris paribus manipulations associated with the effect of some components of \(Y\) on other components of \(Y\) are not possible within the model.
• Consider a two person model of social interactions.
• \( Y_1 \) is the outcome for agent 1;
• \( Y_2 \) is the outcome for agent 2.
•
\[
Y_1 = \alpha_1 + \gamma_{12} Y_2 + \beta_{11} X_1 + \beta_{12} X_2 + U_1 \hspace{1cm} (2a)
\]
\[
Y_2 = \alpha_2 + \gamma_{21} Y_1 + \beta_{21} X_1 + \beta_{22} X_2 + U_2. \hspace{1cm} (2b)
\]
•
\[
E(U_1 \mid X_1, X_2) = 0 \hspace{1cm} (3a)
\]
and
\[
E(U_2 \mid X_1, X_2) = 0. \hspace{1cm} (3b)
\]
• Causal effect of \( Y_2 \) on \( Y_1 \) is \( \gamma_{12} \).
• With no exclusions or *a priori* information cannot identify the causal effect.

• Cowles Monographs 10 and 14 showed how to solve this problem: Assume exclusion ($\beta_{12} = 0$).

• We can identify the *ceteris paribus* causal effect of $Y_2$ on $Y_1$.

• Thus if $\beta_{12} = 0$, from the reduced form

$$\frac{\pi_{12}}{\pi_{22}} = \gamma_{12}.$$ 

• Other restrictions possible.
Counterfactuals, Causality and Structural Econometric Models: Relationship to Pearl’s Analysis

- Controlled variation in external forcing variables is the key to defining causal effects in nonrecursive models.

- It is of some interest to readers of Pearl’s influential book on causality (2000) to compare our use of the standard simultaneous equations model of econometrics in defining causal parameters to his.

- Pearl defines a causal effect by “shutting one equation down” or performing “surgery” in his colorful language.
• He implicitly assumes that “surgery”, or shutting down an equation in a system of simultaneous equations, uniquely fixes one outcome or internal variable.

• In general, it does not.

• Putting a constraint on one equation places a restriction on the entire set of internal variables.

• In general, no single equation in a system of simultaneous equations uniquely determines any single outcome variable.

• Shutting down one equation might also affect the parameters of the other equations in the system and violate the requirements of parameter stability.
• A clearer manipulation that can justify Pearl’s approach but shows its special character is to assume that it is possible to fix $Y_2$ by assuming that it is possible to set $\gamma_{21} = 0$.

• Assume that $U_1$ and $U_2$ are uncorrelated.

• This together with $\gamma_{21} = 0$ makes the model recursive.

• It assumes that agent 1 is unaffected by the consumption of agent 2.
• Under these assumptions, one can regress $Y_1$ on $Y_2, X_1, X_2$ in the population and recover all of the causal parameters.

• Variation in $U_2$ breaks the perfect collinearity among $Y_2, X_1$ and $X_2$.

• In general, as we discuss below, it is often not possible to freely set some parameters without affecting the rest of the parameters of a model.
• Shutting down an equation or fiddling with the parameters in $\Gamma$ is not required to define causality in an interdependent, nonrecursive system or to identify causal parameters.

• The more basic idea is exclusion of different external variables from different equations which, when manipulated, allow the analyst to construct the desired causal quantities.
The Multiplicity of Causal Effects that Can Be Defined from a Simultaneous Equations System

• In the context of the basic nonrecursive model, there are many possible causal variations, richer than what can be obtained from the reduced form.

• Using the reduced form \( Y = X\Pi + \mathcal{E} \), one can define causal effects as \textit{ceteris paribus} effects of variables in \( X \) or \( \mathcal{E} \) on \( Y \).

• This definition solves out for all of the intermediate effects of the internal variables on each other.
• Using the structure (1), one can define the effect of one internal variable on another holding constant the remaining internal variables and \((X, U)\).

• One can, in general, solve out from the general system of equations for a subset of the \(Y\) (e.g., \(Y^*\) where \(Y = (Y^*, Y^{**})\)) using the reduced form of the model.

• Then use \textit{quasi-structural} models to define a variety of causal effects that solve out for some but not all of the possible causal effects of \(Y\) on each other.
• These quasi-structural models may be written as

\[ \Gamma^{**} Y^{**} = \Pi^{**} X + U^{**}. \]

• This expression is obtained by using the reduced form for component

\[ Y^* : Y^* = \Pi^* X + \mathcal{E}^* \]

and substituting for \( Y^* \) in (1).

• \( U^{**} \) is the error term associated with this representation.

• There are many possible quasi-structural models.
Counterfactuals, Causality and Structural Econometric Models: Structure as Invariance to a Class of Modifications

- A basic definition of a system of structural relationships is that it is a system of equations invariant to a class of modifications or interventions.

- In the context of policy analysis, this means a class of policy modifications.

- This is the definition proposed by Hurwicz (1962).

- It is implicit in Marschak (1953) and it is explicitly utilized by Sims (1977), Lucas and Sargent (1981), and Leamer (1985), among others.
• The mechanisms generating counterfactuals and the choices of counterfactuals have already been characterized.

• Policies can act on preferences and the arguments of preferences (and hence choices), on outcomes $Y(s, \omega)$ and the determinants affecting outcomes or on the information facing agents.

• Recall that $g_s, s \in S$, generates outcomes while $f_s, s \in S$, generates subjective evaluations.
Specifically,

Policies can shift the distributions of outcomes and choices 
\((Q, Z, X, U, η)\), where 
\(Q = \{Q(s, ω)\}_{s \in S}\), 
\(Z = \{Z(s, ω)\}_{s \in S}\), 
\(η = \{η(s, ω)\}_{s \in S}\), and 
\(U = \{U_s(ω)\}_{s \in S}\) in the population. This may entail defining the \(g_s\) and \(f_s\) over new domains. Let \(\mathcal{X} = (Q, Z, X, U, η)\) be sets of arguments of the determinants of outcomes. Policies shifting the distributions of these variables are characterized by maps \(T_\mathcal{X} : \mathcal{X} \mapsto \mathcal{X}'\).
Policies can select new $f$, $g$, or $\{f_s, g_s\}_{s \in S}$ functions. In particular, new arguments (e.g., amenities or characteristics of programs) may be introduced as a result of policy actions creating new attributes. Policies shifting functions map $f$, $g$, or $\{f_s, g_s\}_{s \in S}$ into new functions $T_f : f_s \mapsto f'_s$; $T_g : g_s \mapsto g'_s$. This may entail changes in functional forms with a stable set of arguments as well as changes in arguments of functions.

Policies may affect individual information sets $(I_\omega)_{\omega \in \Omega}$. $T_{I_\omega} : I_\omega \mapsto I'_\omega$. 
• Clearly, any particular policy may incorporate elements of all three types of policy shifts.

• Parameters of a model or parameters derived from a model are said to be policy invariant with respect to a class of policies if they are not changed (are invariant) when policies within the class are implemented.

• More generally, policy invariance for \( f, g \) or \( \{f_s, g_s\}_{s \in S} \) requires for a class of policies \( \mathcal{P}_A \subseteq \mathcal{P} \):

\[
(\text{PI-5}) \quad \text{The functions } f, g, \text{ or } \{f_s, g_s\}_{s \in S} \text{ are the same for all values of the arguments in their domain of definition no matter how their arguments are determined, for all policies in } \mathcal{P}_A.
\]
• This definition is a version of (PI-3) and (PI-4) for the specific notation of the choice model developed in this presentation and for specific types of policies.
• In the econometric approach to policy evaluation, the analyst attempts to model how a policy shift affects outcomes without reestimating any model.

• Thus, for the tax and labor supply example presented above, with labor supply function \( h = h(w(1 - s), x, u_s) \), it is assumed that we can shift tax rate \( s \) without affecting the functional relationship mapping \( (w(1 - s), x, u_s) \) into \( h \).

• If, in addition, the support of \( w(1 - s) \) under one policy is the same as the support determined by the available economic history, for a class of policy modifications (tax changes), the labor supply function can be used to accurately predict the outcomes for that class of tax policies.
• In the simultaneous equations model analyzed above, invariance requires stability of $\Gamma$, $B$ and $\Sigma_U$ to interventions.

• Policy invariant parameters are not necessarily causal parameters as we noted in our analysis of reduced forms.

• Thus, in the simultaneous equations model, depending on the \textit{a priori} information available, it may happen that no causal effect of one internal variable on another may be defined but if $\Pi$ is invariant to modifications in $X$, the reduced form is policy invariant for those modifications.
• The class of policy invariant parameters is thus distinct from the class of causal parameters, but invariance is an essential attribute of a causal model.

• For counterfactuals $Y(s, \omega)$, if assumption (PI-3) is not postulated for a class of policies $\mathcal{P}_A$, all of the treatment effects defined above would be affected by policy shifts.
• Rubin’s SUTVA assumptions (R-3) and (R-2) are versions of Hurwicz’s (1962) invariance assumptions for the objective outcomes.

• Thus Rubin’s assumption (R-3) postulates that $Y(s, \omega)$ is invariant to all policies that change $f$ but does not cover policies that change $g$ or the support of $Q$.

• “Deep structural” parameters generating the $f$ and $g$ are invariant to policy modifications that affect technology, constraints and information sets except when the policies extend the historical supports.
Counterfactuals, Causality and Structural Econometric Models: Alternative Definitions of “Structure”

• The terms “structural equation” or “structure” are used differently by different analysts and are a major source of confusion in the policy analysis literature.

• We briefly distinguish three other definitions of structure besides our version of Hurwicz (1962).
The traditional Cowles Commission structural model of econometrics was presented above.

It is a nonrecursive model for defining and estimating causal parameters.

It is also a framework for relaxing assumptions (PI-3) and (PI-4).
• It is a useful vehicle for distinguishing effects that can be defined in principle (through *a priori* theory) from effects that are identifiable from data.

• This is the contrast between tasks 1 and 2 of table 1.

• The framework arose as a model to analyze the economic phenomenon of supply and demand in markets, and to analyze policies that affected price and quantity determination.
Table 1: Three distinct tasks arising in the analysis of causal models

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• A second definition of structure, currently the most popular in the applied economics literature, defines an equation or a system of equations as structural if they are derived from an explicitly formulated economic theory.

• Consider a consumer demand problem where a consumer $\omega$ chooses among goods $X(\omega)$ given money income $M(\omega)$ and prices $P$, $P'X(\omega) \leq M(\omega)$.

• Preferences of $\omega$, $R(X, \omega)$, are quasiconcave in $X(\omega)$ and twice differentiable.

• Many economists would say that $R(X, \omega)$ is structural because it describes the preferences of agent $\omega$. 
• When we solve for the demand functions, under standard conditions, we obtain

\[ X = X \left( \frac{P}{M}, \omega \right). \]

• These are sometimes called “reduced form” expressions by analogy with the Cowles Commission simultaneous equations literature expositied above, assuming that prices normalized by income are exogenous.
• While any convention is admissible, this one is confusing since we can recover the preferences (up to a monotonic function) given the demand function under standard regularity conditions (see, e.g., Varian, 1978).

• Is the indirect utility function

\[ \tilde{R}^*(\omega, \frac{P}{M}) = R(X \left( \frac{P}{M} \right), \omega) = R^* \left( \frac{P}{M}, \omega \right) \]

structural or reduced form?
• While the notion of structure in this widely applied usage is intuitively clear, it is not the same notion of structure as used in Cowles Commission econometrics as defined above.

• It is structural in the sense that the internal variables (the $X$ in this example) are substituted out for externally specified (to the consumer) $P$ and $M$.

• At the market level, this distinction is not clear cut since $X$ and $P$ are jointly determined.
• The notion of a “reduced form” is not clearly specified until the statistical properties of $X$, $P$ or $M$ have been specified.

• Recall that the Cowles Commission definition of reduced form

  1 solves out the $Y$ in terms of $X$, and

  2 assumes that $X$ is “exogenous” relative to $U$.

• In current popular usage, a reduced form makes both assumptions.
• A third definition of a structural model is as a finite parameter model.

• Thus in our consumer demand example, if we parameterize $R(X, \omega)$ by a finite dimensional vector $\theta$, we may write $R(X, \omega) = R(X; \theta)$.

• This notion of structural is often used in statistics.

• Applied to a demand system, it would write $X \left( \frac{P}{M}, \omega \right) = X \left( \frac{P}{M}; \tilde{\theta} \right)$ where $\tilde{\theta}$ is a finite parameter vector.

• Structural in this sense means low dimensional and is not related to the endogeneity of any variable or the economic interpretation placed on the equations.
A more basic definition of a system of structural equations, and the one featured in this chapter, is a system of equations invariant to a class of modifications.

Without such invariance one cannot trust the models to forecast policies or make causal inferences.

Invariance to modifications requires a precise definition of a policy, a class of policy modifications and specification of a mechanism through which policy operates.
Counterfactuals, Causality and Structural Econometric Models: Counterfactuals, Causality and Structural Econometric Models

- Economically well-posed models make explicit the assumptions used by analysts regarding preferences, technology, the information available to agents, the constraints under which they operate, and the rules of interaction among agents in market and social settings and the sources of variability among agents.
• These explicit features make these models, like all scientific models, useful vehicles for

1. interpreting empirical evidence using theory,

2. collating and synthesizing evidence across studies using economic theory,

3. measuring the welfare effects of policies, and

4. forecasting the welfare and direct effects of previously implemented policies in new environments and the effects of new policies.
• These features are absent from the modern treatment effect literature.

• At the same time, this literature makes fewer statistical assumptions in terms of exogeneity, functional form, exclusion and distributional assumptions than the standard structural estimation literature in econometrics.

• These are the attractive features of this approach.
To reconcile the econometric and treatment effect literatures, go back to a neglected but important paper by Marschak (1953) and taught in his 1949 lectures at Chicago in the Cowles Commission.

Marschak noted that for many specific questions of policy analysis, it is not necessary to identify fully specified economic models that are invariant to classes of policy modifications.

Implicit was his use of what we would now call decision theory.
• All that may be required for certain policy analyses are combinations of subsets of the structural parameters, corresponding to the parameters required to forecast particular policy modifications, which are often much easier to identify (i.e., require fewer and weaker assumptions).

• Forecasting or evaluating policies may only require partial knowledge of the full simultaneous equations system.

• This principle called Marschak’s maxim in honor of this insight.
• The modern statistical treatment effect literature implements Marschak’s maxim where the policies analyzed are the treatments available under a particular policy regime $p \in \mathcal{P}$.

• The goal of policy analysis under this approach is typically restricted to evaluating policies in place and not in forecasting the effects of new policies or the effects of old policies on new environments.
• What is often missing from the literature on treatment effects is a clear discussion of the economic question being addressed by the treatment effect being estimated.

• This is the unstated and hence the unanswered question in the literature.

• When the treatment effect literature does not clearly specify the economic question being addressed, it does not implement Marschak’s maxim.
• Population mean treatment parameters are often identified under weaker conditions than are traditionally assumed in structural econometric analysis.

• Thus to identify the average treatment effect for $s$ and $s'$ we only require

$$E(Y(s, \omega) \mid S = s, X = x) - E(Y(s', \omega) \mid S = s', X = x).$$

• Do not need exogeneity of $X$.

• The parameter is not designed to evaluate a whole host of other policies.
• Viewed in this light, the treatment effect literature that compares the outcome associated with \( s \in S \) with the outcome associated with \( s' \in S \) seeks to recover a causal effect of \( s \) relative to \( s' \).

• It is a particular causal effect for a particular set of policy interventions.

• It is structural for this intervention.

• Marschak’s maxim urges analysts to formulate the problem being addressed clearly and to use the minimal ingredients required to solve it.
• The treatment effect literature addresses the problem of comparing treatments \( s \in S \) under policy regime \( p \in \mathcal{P} \), for a particular environment.

• As analysts ask more difficult questions, it is necessary to specify more features of the models being used to address the questions.

• Marschak’s maxim is an application of Occam’s Razor to policy evaluation.
• For certain classes of policy interventions designed to answer problem (P-1), the treatment effect approached may be very powerful and more convincing than explicitly economically formulated models because they entail fewer assumptions.

• However, considerable progress has been made in relaxing the parametric structure assumed in the early explicitly economic models.
• As the treatment effect literature is extended to address the more general set of policy forecasting problems entertained in the explicitly economic literature, the distinction between the two approaches will vanish.

• To make these methods empirically operational, we need to investigate the identification problem.

• This is task 2 in table 1.
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