Some implications of (non-)ergodicity of psychological processes

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Standard approach to statistical analysis in psychology:

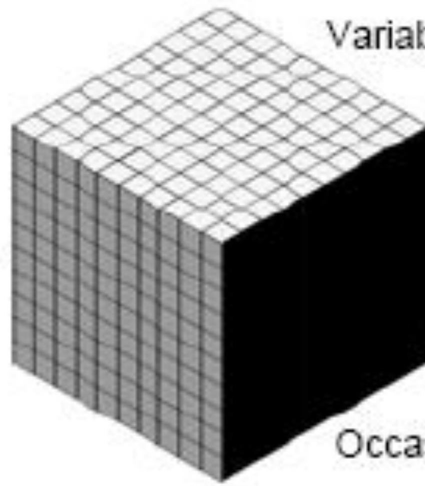
- Analysis of **inter**-individual variation (variation **between** subjects in a population of subjects; individual differences)

- Strong assumption of **homogeneity** in (sub-)populations

- Aimed at generalization to the state of affairs at the **population level**

- **Implicit** assumption of applicability of results at the individual level of **intra**-individual variation

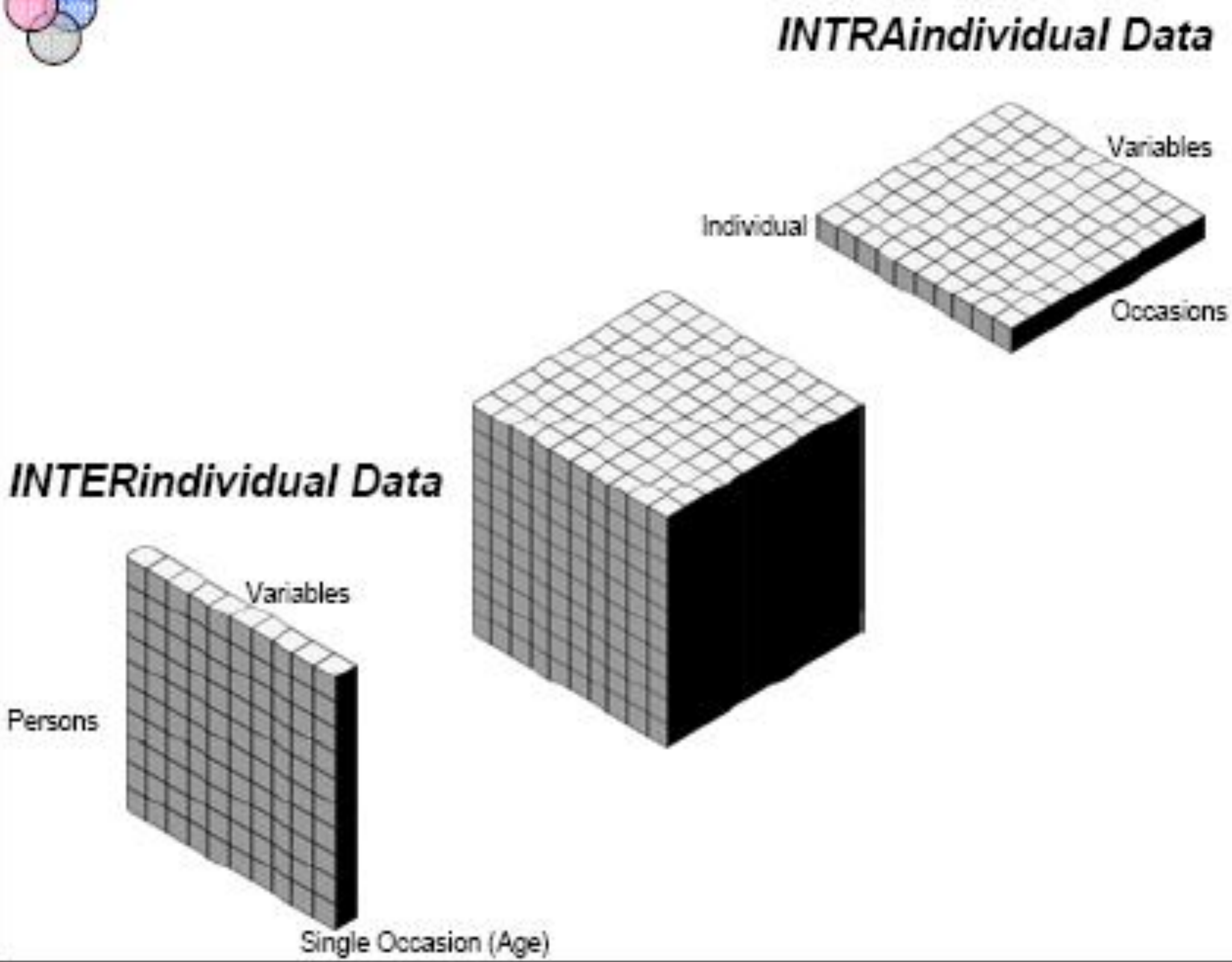
The Data Box *(Cattell, 1952)*



Datum

= score for person *i* on
variable *x* at occasion *t*

Nesselroade & Ram, 2004
Ram & Nesselroade, 2007


INTRAindividual Data
Variables
Individual
Occasions
INTERindividual Data
Variables
Persons
Single Occasion (Age)

R-technique: Analysis of **inter**-individual variation

Variables: fixed

Persons: random

Occasions: fixed

Pooling/Generalization across Persons

P-technique: Analysis of **intra**-individual variation

Variables: fixed

Person(s): fixed

Occasions: random

Pooling/Generalization across Occasions

Basic Question:

Can results obtained in analyses of **inter**-individual variation be validly generalized to the subject-specific level of **intra**-individual variation (and vice versa)?

Molenaar, Measurement, 2004

Definition: A process is **non-ergodic** in case results of analysis of inter-individual variation do **not** generalize to the level of intra-individual change in time, and vice versa

Equivalently: A process is **ergodic** in case results of analysis of inter-individual variation validly generalize to the level of intra-individual change in time, and vice versa

Ergodicity is **not** a generic property of homogeneous Hamiltonian systems (the primary class of candidate ergodic dynamic systems).

Ergodicity is the **weakest** property in the ergodic hierarchy, including mixing and K-systems.

Emch, G., & Liu, C. (2002). The logic of thermo-statistical physics. Berlin: Springer.

Theorem (based on Birkhoff, 1931): A Gaussian process is **non-ergodic** if it is:

- **heterogeneous in time** $\rightarrow$ **non-stationary**
  (time-varying trends, variances, etc.)

and/or

- **heterogeneous across subjects**
  (subject-specific dynamics)

Model heterogeneity across subjects (second criterion in Theorem) is invisible in standard factor analysis of inter-individual variation.

Formal proof in: Kelderman & Molenaar (2007) *Multivariate Behavioral Research*, *42*, 435-456.

In general, whether or not a Gaussian process is non-ergodic involves tests whether:

a) the process is nonstationary (first criterion based on Birkhoff’s theorem) and/or
b) whether the process has subject-specific dynamics (second criteron based on Birkhoff’s theorem).

These tests involve dedicated analyses of **intra**-individual variation (time series analysis).

Immediate Consequence of Theorem:

Developmental and Learning Processes have time-varying statistical characteristics, hence are heterogeneous in time (non-stationary).

Consequently these processes are **non-ergodic** and their analysis has to be based on time series of **intra**-individual change (time series analysis).

Immediate Consequence of Theorem:

Classical Test Theory is non-ergodic. In particular, the intra-individual means and variances (test reliabilities) are different from the inter-individual mean and variance (test reliability).

Molenaar, P.C.M. (2008). In: S.M. Hofer & D.F. Alwin (Eds.), *Handbook of cognitive aging*. Thousand Oaks: Sage, 90-104.

Testing heterogeneity across subjects (second criterion based on Birkhoff’s theorem)

Person-Specific Factor Models

Molenaar & Campbell, Current Directions in Psychology, 2009
Hamaker, Dolan, & Molenaar, Multivariate Behavioral Research, 2005
Timmerman, Dissertation, 2001

Application to Big Five Factor Model data:

Borkenau, P., & F. Ostendorf, (1998). The Big Five as states: How useful is the five-factor model to describe intra-individual variations over time? *Journal of Personality Research*, *32*, 202-221.

Replicated time series design:

N=22 subjects measured at T=90 consecutive days with the same questionnaire composed of 30 items (6 per personality factor).

Model of the Relationship Between Specific Traits and the Big Five Factors of Personality for the General Population (A) and Intraindividual Models for Three Representative Subjects (B, C, and D)

	Interindividual model (A)					Intraindividual models (3 subjects)								
						(B)			(C)				(D)	
Trait adjective	Neuroticism (N)	Extraversion (E)	Agreeableness (A)	Conscientiousness (C)	Intellect (I)	F1	F2	F3	F1	F2	F3	F4	F1	F2
irritable	x					x				x			x	
vulnerable	x					x				x			x	
emotionally stable	x					x				x			x	
calm	x					x				x			x	
resistant	x					x				x			x	
changeable	x					x			x				x	
dynamic		x						x						
sociable		x						x					x	
lively		x						x				x		
shy		x						x						
silent		x						x				x	x	
reserved		x						x				x		
bad-tempered						x			x				x	
good-natured			x			x			x				x	
helpful			x						x					
considerate			x			x			x				x	
selfish			x			x								
domineering			x			x			x					
obstinate			x			x			x					
industrious				x			x							x
persistent				x										
responsible				x					x					
lazy				x			x		x					x
reckless									x					
witty					x						x			
knowledgeable					x		x				x			x
prudent					x		x				x			
fanciness					x						x			
uninformed					x		x				x			
unimaginative					x		x				x			x

Heterogeneity: Nonlinear Epigenetics

$$\partial u/\partial t = \rho u^2/v - \beta_u u + \sigma_u + D_u \partial^2 u/\partial x^2$$

$$\partial v/\partial t = \rho v^2 - \beta_v v + \sigma_v + D_v \partial^2 v/\partial x^2$$

Molenaar, P.C.M., Boomsma, D.I., & Dolan, C.V. (1993). A third source of developmental differences. *Behavior Genetics*, *23*, 519-524.

Molenaar, P.C.M., & Raijmakers, M.E.J. (1999). Additional aspects of third source variation for the genetic analysis of human development and behavior.*Twin Research*, *2*, 49-52.

Kan et al. (2010), Nonlinear epigenetic variance: Review and simulations. Developmental Science, 13, 11-27.

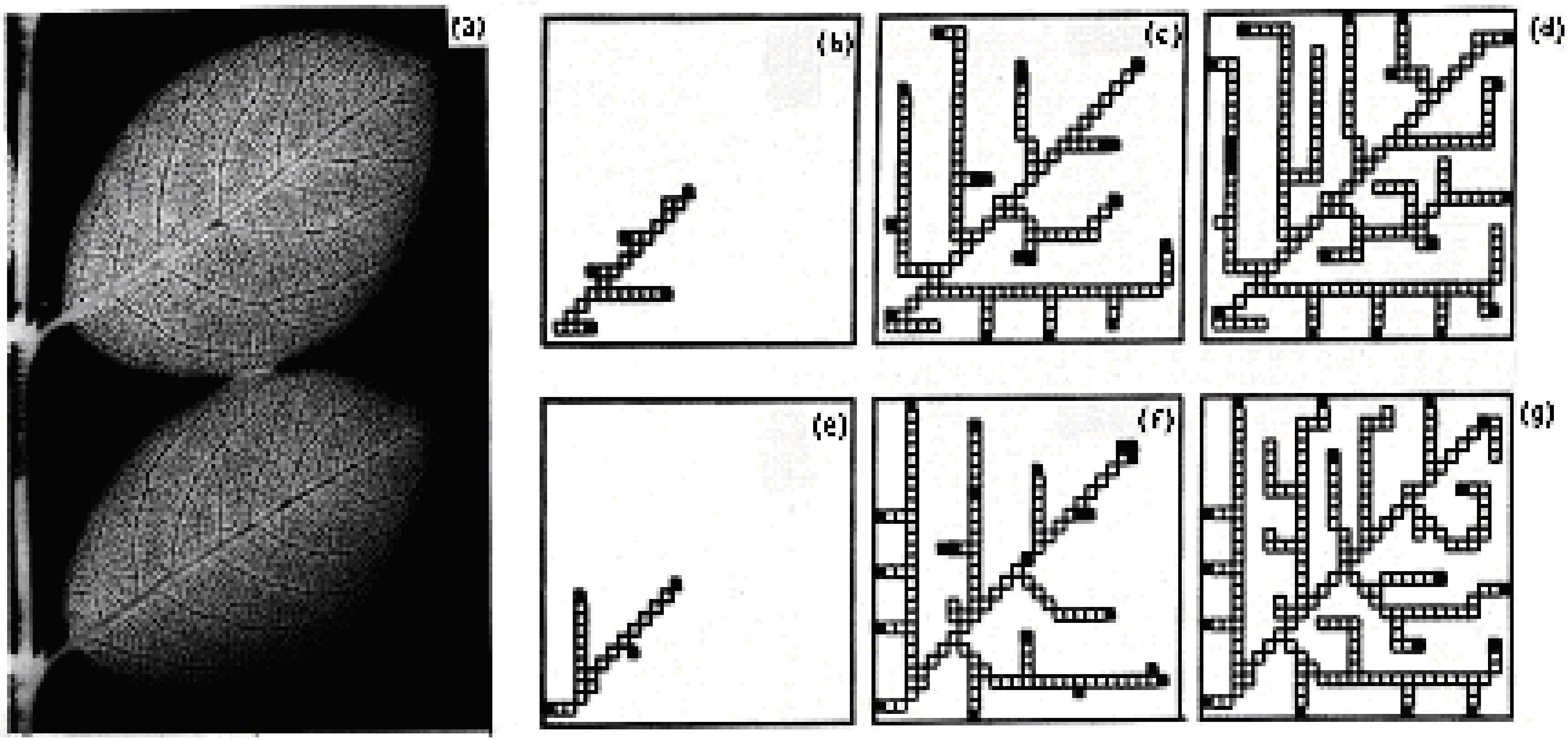


Fig. 15.5. Influence of random fluctuation on pattern formation. (a) Two leaves of the same tree. Their pattern formation is presumably controlled by the same genetic information. Nevertheless, the pattern is only similar, not identical. (b–d) and (e–g) Two simulations with the same parameters and the same initial condition but with different random fluctuation (3%) in the constant c, eq. 15.1a,b. The model reaction determines only properties of the overall pattern such as average net density. Fine details are influenced by small local differences.

Heterogeneity: Functional brain connectivity

The functional neural networks underlying brain processes differ dramatically across individuals throughout the life span and therefore their analysis should be based on intra-individual variation, not group data.

Nelson, C.A., de Haan, M., & Thomas, K.M. (2006). *Neuroscience of cognitive development: the role of experience and the developing brain*. New York: Wiley.

Sporns, O. (2010). *Networks of the brain*. Cambridge, Mass.: MIT Press.

Heterogeneity: Human decision making

"Each axiom should be tested in as much isolation as possible, and it should be done in-so-far as possible for each person individually, not using group aggregations."

Luce, R.D. (2000). *Utility of gains and losses: Measurement-theoretical and experimental approaches*. Mahwah, NJ, Erlbaum, p.29

"Extremely large individual differences" in binary choice behavior.

Erev, I., & Barron, G. (2005). On adaptation, maximization, and reinforcement learning among cognitive strategies. *Psychological Review*, *112*, 912-931.

Ergodic Longitudinal Factor Model with Time-Varying Covariates

$\mathbf{y}_i(t) = \boldsymbol{\Lambda}\boldsymbol{\eta}_i(t) + \boldsymbol{\varepsilon}_i(t)$

$\boldsymbol{\eta}_i(t) = \alpha + \mathbf{B}\boldsymbol{\eta}_i(t\text{-}1) + \Gamma\mathbf{u}_i(t\text{-}1) + \boldsymbol{\zeta}_i(t)$

t=1,...,T; T fixed; i=1,…,N; N random

Additional assumptions for LFM:

Homogeneity across subjects (equal dynamics)
Gaussian distributions

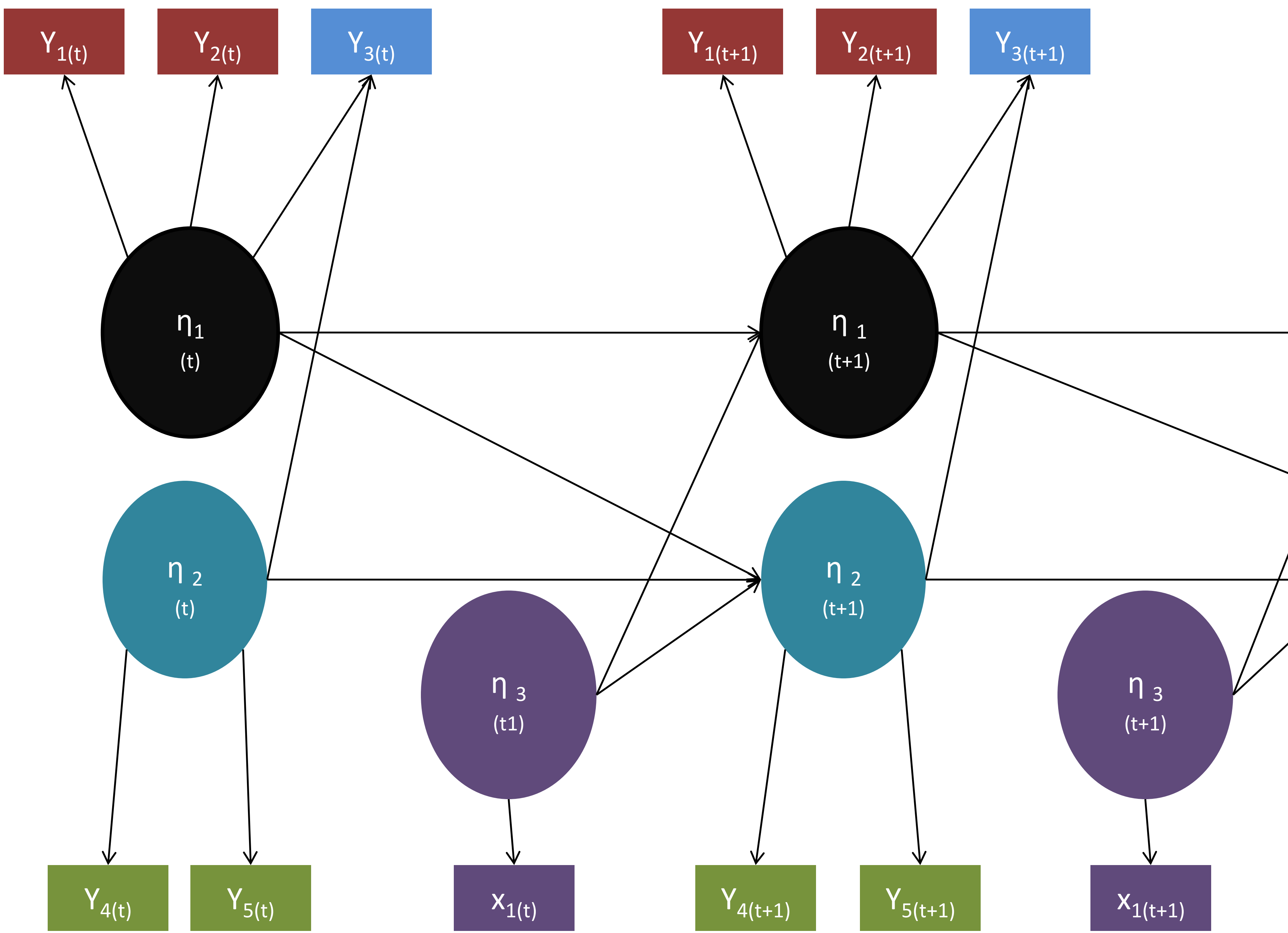
Computational LQG Control Goal:

Determine the covariable $\mathbf{u}_i(t)$ in such a way that for each subject i = 1,…,N the longitudinal factor scores $\boldsymbol{\eta}_i(t+1)$ at the next time point t+1 are as closely as possible to their desired values while minimizing the cost of manipulation of $\mathbf{u}_i(t)$.

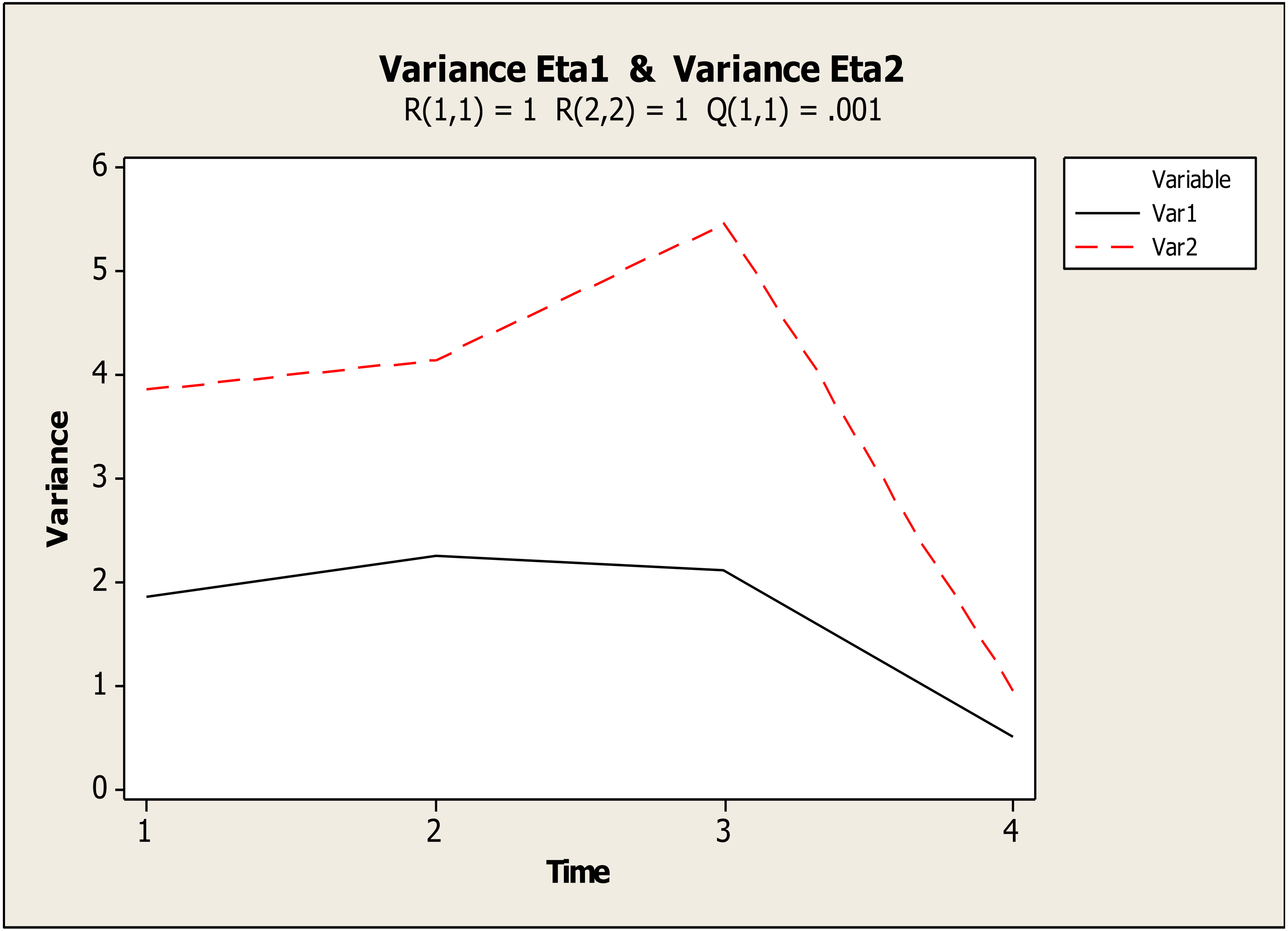
Molenaar, (2010), J. Math. Psych., 54, 208-213.

Application to Ergodic Longitudinal Factor Model with 5 y-variables and 1 time-varying covariable measured at 3 measurement occasions

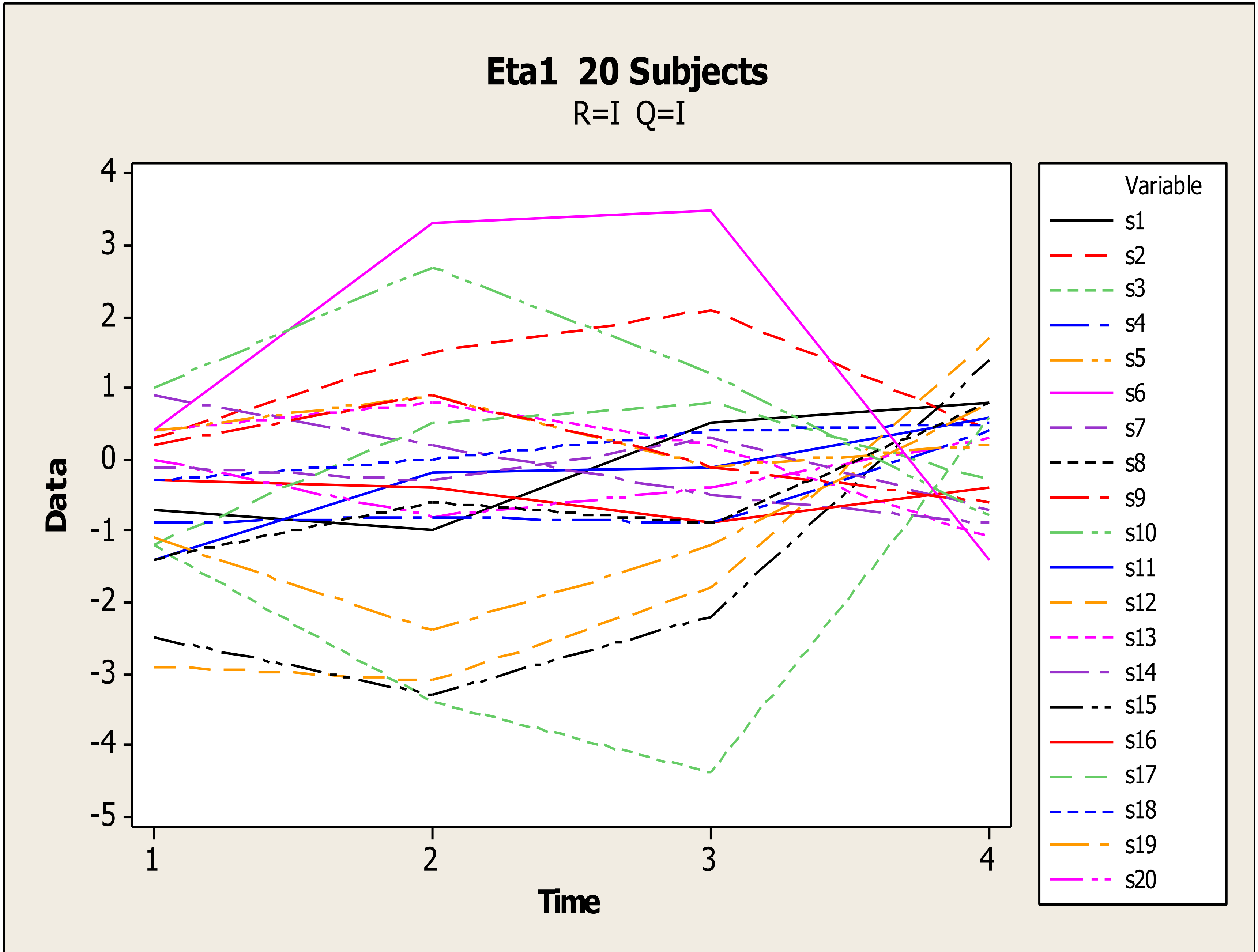
Control goal is to steer the centered factor scores $\boldsymbol{\eta}_i(4)$ for each subject i = 1,…, 300 at the next (fourth) measurement occasion to zero (reduce the variance of the factor scores).

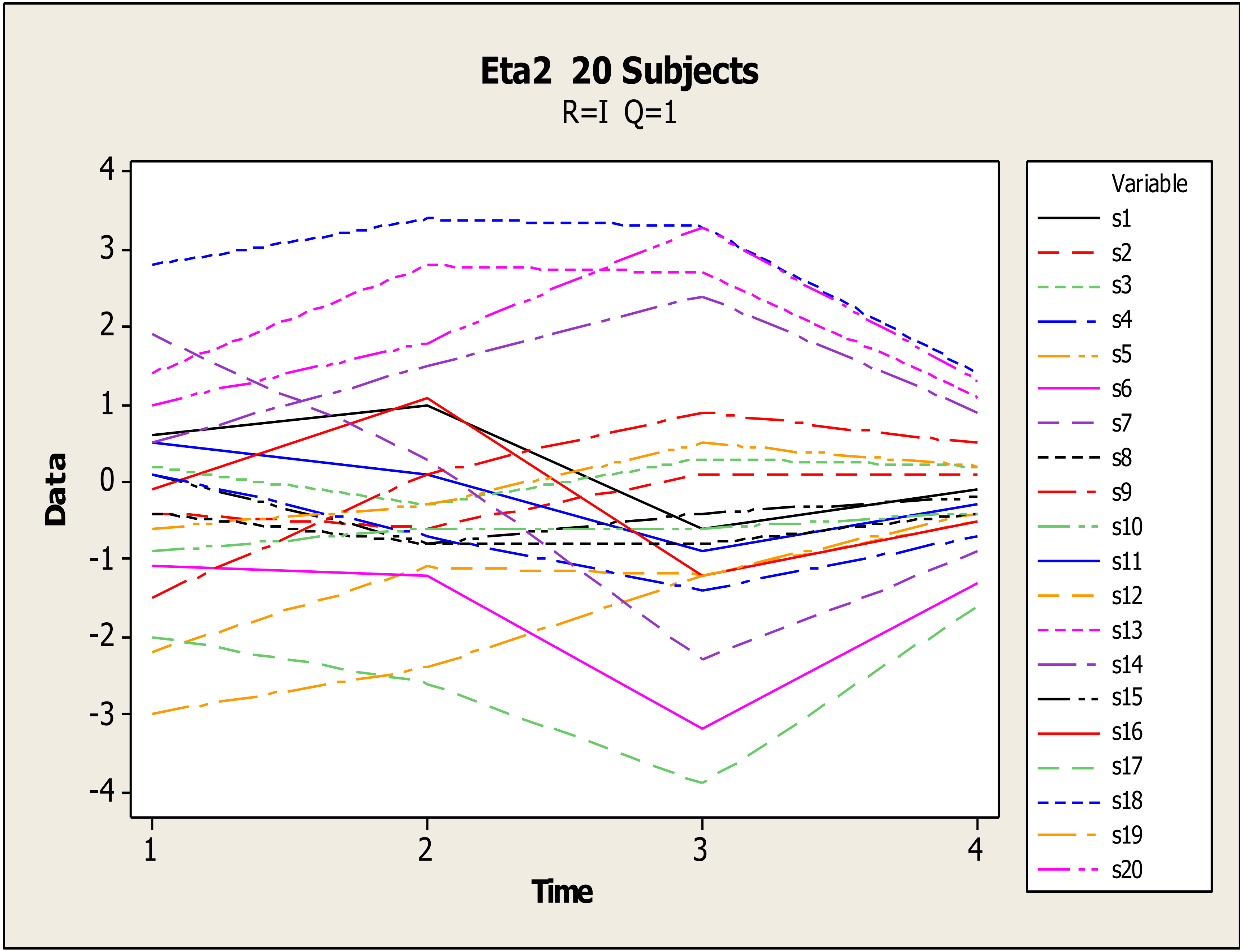


Y1(t)
Y2(t)
Y3(t)
Y1(t+1)
Y2(t+1)
Y3(t+1)
η1
(t)
η 1
(t+1)
η 2
(t)
η 2
(t+1)
η 3
(t1)
η 3
(t+1)
Y4(t)
Y5(t)
x1(t)
Y4(t+1)
Y5(t+1)
x1(t+1)

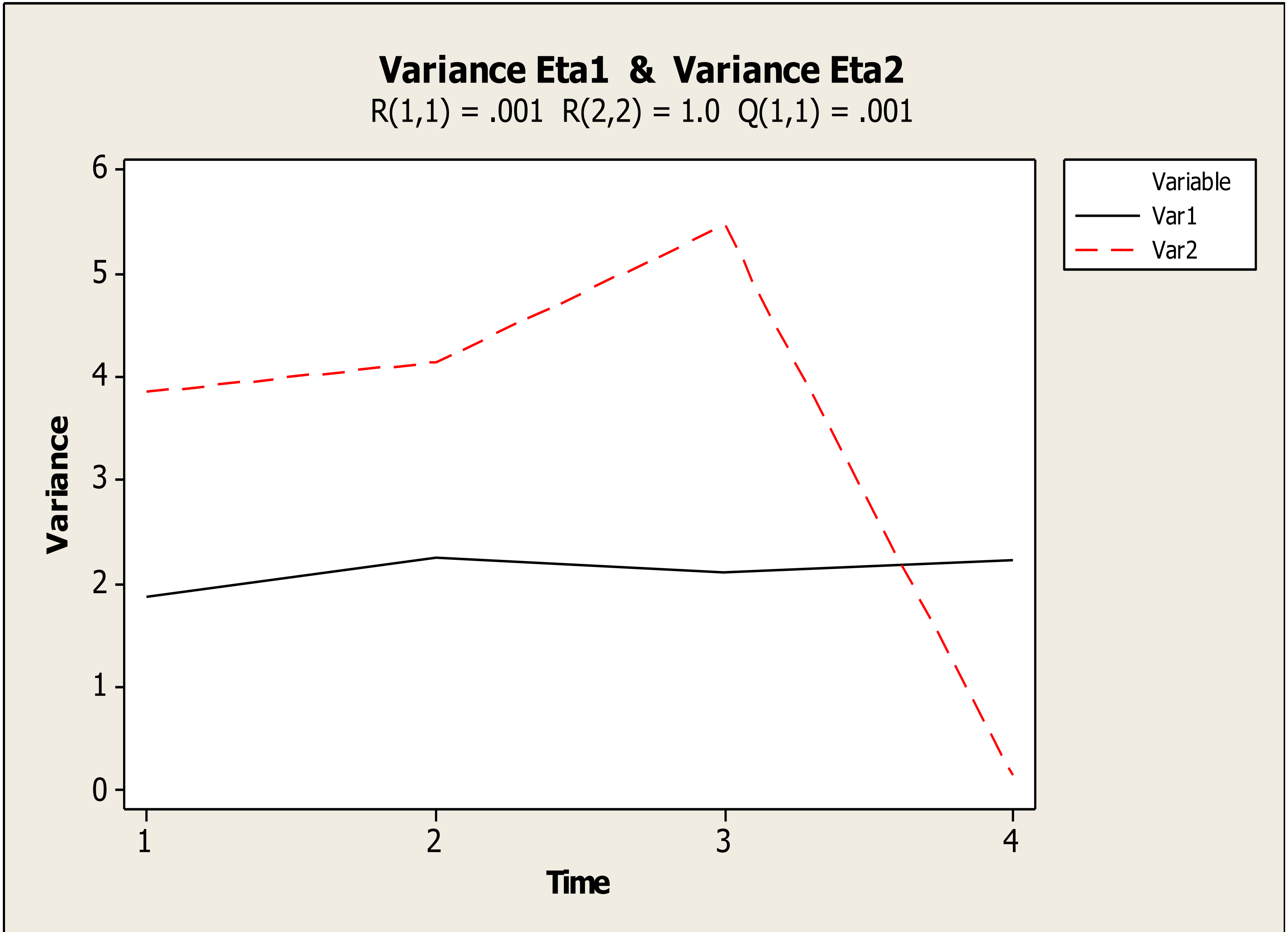


Variance Eta1 & Variance Eta2
R(1,1) = 1 R(2,2) = 1 Q(1,1) = .001
Variance
0
1
2
3
4
5
6
1
2
3
4
Time
Variable
Var1
Var2


Eta1 20 Subjects
R=I Q=I
Data
Time
4
3
2
1
0
-1
-2
-3
-4
-5
1
2
3
4
Variable
s1
s2
s3
s4
s5
s6
s7
s8
s9
s10
s11
s12
s13
s14
s15
s16
s17
s18
s19
s20



Eta2 20 Subjects
R=I Q=1
Data
Time
4
3
2
1
0
-1
-2
-3
-4
1
2
3
4
Variable
s1
s2
s3
s4
s5
s6
s7
s8
s9
s10
s11
s12
s13
s14
s15
s16
s17
s18
s19
s20


Variance Eta1 & Variance Eta2
R(1,1) = .001 R(2,2) = 1.0 Q(1,1) = .001
Variance
Time
0
1
2
3
4
5
6
1
2
3
4
Variable
Var1
Var2